Maths

09 Jan. 2019 [Session 2:30 PM to 5:30 PM]

RED COLOUR CONSIDER OFFICIAL ANSWER

1. Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to :

माना a, b तथा c एक समान्तर श्रेढ़ी (जो कि अचर समान्तर श्रेढ़ी नहीं है) के क्रमशः 7^{th}, 11^{th} तथा 13^{th} पद है। यदि ये एक गुणोत्तर श्रेढ़ी के भी तीन क्रमागत पद है तो $\frac{a}{c}$ बराबर है –

(1) $\frac{7}{13}$ (2) 2 (3) 4 (4) $\frac{1}{2}$

A. 3

Sol. $(T_{11})^2 = (T_7)(T_{13})$

$\Rightarrow (a+10d)^2 = (a+6d)(a+12d)$

$\Rightarrow a = -14d$

$\frac{T_7}{T_{13}} = \frac{a+6d}{a+12d} = \frac{8}{2} = 4$

Question ID : 4165298853

Option 1 ID : 41652934872 Option 2 ID : 41652934871

Option 3 ID : 41652934870 Option 4 ID : 41652934873

2. Let $A = \{x \in R : x \text{ is not a positive integer}\}$. Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$, then f is :

(1) not injective (2) surjective but not injective

(3) injective but not surjective (4) neither injective nor surjective

माना $A = \{x \in R : x$ एक धन पूर्णांक नहीं है$\}$. फलन $f : A \to R$ निम्न प्रकार से परिभाषित है $f(x) = \frac{2x}{x-1}$, तो f एक :

(1) एकैकी फलन नहीं है। (2) आच्छादक है, परन्तु एकैकी फलन नहीं है।

(3) एकैकी है, परन्तु आच्छादक फलन नहीं है (4) न एकैकी है और न आच्छादक फलन है

A. 3

Sol. $f(x) = \frac{2x}{x-1} = 2 + \frac{2}{x-1}$, $f'(x) = \frac{-2}{(x-1)^2} < 0 \; \forall \; x \in \text{Domain}$

$y = \frac{2x}{x-1} \Rightarrow xy - y = 2x \Rightarrow x = \frac{y}{y-2} \neq I^+$

So Range can not be R

Question ID : 4165298846

Option 1 ID : 41652934842 Option 2 ID : 41652934844

Option 3 ID : 41652934843 Option 4 ID : 41652934845

3. The logical statement $[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ is equivalent to :

तर्कसंगत कथन $[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ निम्न में से किसके समतुल्य है –

(1) $(p \wedge \sim q) \vee r$ (2) $(p \wedge r) \wedge \sim q$ (3) $(\sim p \wedge \sim q) \wedge r$ (4) $\sim p \vee r$

A. 2

Sol. $\left[\sim(\sim p \vee q) \wedge (p \wedge r)\right] \wedge (-q \wedge r)$

$= \left[(p \wedge \sim q) \vee (p \wedge r)\right] \wedge (\sim q \wedge r)$

$= \left[p \wedge (\sim q \vee r)\right] \wedge (\sim q \wedge r)$

$= p \wedge (\sim q \wedge r)$

$= (p \wedge r) \wedge (-q)$

Question ID : 4165298875

Option 1 ID : 41652934960 **Option 2 ID : 41652934959**

Option 3 ID : 41652934961 Option 4 ID : 41652934958

4. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is :

α के उन सभी संभावित धन पूर्णांक मानों की संख्या जिनके लिए द्विघातीय समीकरण $6x^2 - 11x + \alpha = 0$ के मूल परिमेय संख्यायें है, है –

(1) 2 (2) 3 (3) 4 (4) 5

A. 2

Sol. $x = \dfrac{11 \pm \sqrt{121 - 24\alpha}}{12}$

$121 - 24\alpha$ Should be perfect square

So $\alpha = 3, 5, 4$

Question ID : 4165298848

Option 1 ID : 41652934850 **Option 2 ID : 41652934851**

Option 3 ID : 41652934852 Option 4 ID : 41652934853

5. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repetition of digits allowed) is equal to :

अंकों 0,1,3,7,9 के प्रयोग से (जहां अंकों को दोहराया जा सकता है) बनाई जा सकने वाली प्राकृत संख्यायें जो 7,000 से कम है, की संख्या है –

(1) 374 (2) 250 (3) 375 (4) 372

A. 1

Sol. 4 digit numbers less than 7000 = $2 \times 5 \times 5 \times 5 = 250$

3 digit numbers = $4 \times 5 \times 5$

2 digit numbers = 4×5

Single digit = 4

Total numbers = 375

Question ID : 4165298851

Option 1 ID : 41652934864 Option 2 ID : 41652934865

Option 3 ID : 41652934863 Option 4 ID : 41652934862

6. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then :

यदि रेखायें $x = ay + b$, $z = cy + d$ तथा $x = a'z + b'$, $y = c'z + d'$ लम्बवत् है, तो –

(1) $ab' + bc' + 1 = 0$ (2) $cc' + a + a' = 0$ (3) $aa' + c + c' = 0$ (4) $bb' + cc' + 1 = 0$

A. 3

Sol. $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c},\frac{x-b'}{a'}=\frac{y-d'}{c'}=\frac{z}{1}$

$\Rightarrow \left(a\hat{i}+\hat{j}+c\hat{k}\right).\left(a'\hat{i}+c'\hat{j}+\hat{k}\right)=0$

$\Rightarrow a.a'+c+c'=0$

Question ID : 4165298868

Option 1 ID : 41652934930 Option 2 ID : 41652934933

Option 3 ID : 41652934932 Option 4 ID : 41652934931

7. If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to :

यदि $x = \sin^{-1}(\sin 10)$ तथा $y = \cos^{-1}(\cos 10)$ है, तो $y - x$ बराबर है –

(1) 10 (2) 7π (3) 0 (4) π

A. 4

Sol. $x = \sin^{-1}(\sin(10)) = 3\pi - 10$

$y = \cos^{-1}(\cos(10)) = 4\pi - 10$

$y - x = \pi$

Question ID : 4165298874

Option 1 ID : 41652934956 Option 2 ID : 41652934957

Option 3 ID : 41652934954 **Option 4 ID : 41652934955**

8. A data consists of n observations :

$x_1, x_2, \ldots\ldots, x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then standard deviation of this data is :

आंकड़ों के एक समूह में n प्रेक्षण $x_1, x_2, \ldots\ldots, x_n$ है।

यदि $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ तथा $\sum_{i=1}^{n}(x_i-1)^2 = 5n$ है तो इन आंकड़ों का मानक विचलन है –

(1) $\sqrt{5}$ (2) 5 (3) $\sqrt{7}$ (4) 2

A. 1

Sol. $\left.\begin{aligned}\Sigma(x_i+1)^2 = 9n = \Sigma x_i^2 + 2\Sigma x_i + \Sigma.1\end{aligned}\right\} \Rightarrow \Sigma x_i^2 = 6n$

$\left.\begin{aligned}\Sigma(x_i-1)^2 = 5n = \Sigma x_i^2 - 2\Sigma x_i + \Sigma.1\end{aligned}\right\} \Rightarrow \Sigma x_i = n$

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum x_i^2 - 2(\sum x_i)\bar{x} + \sum \bar{x}}{n}} = \sqrt{\frac{6n - 2n + n}{n}} = \sqrt{5}$$

$$\bar{x} = \frac{\sum x_i}{n} = 1$$

Question ID : 4165298871

Option 1 ID : 41652934943 Option 2 ID : 41652934944

Option 3 ID : 41652934942 Option 4 ID : 41652934945

9. For each $x \in R$, let [x] be the greatest integer less than or equal to x. Then $\lim\limits_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

सभी $x \in R$ के लिए, माना [x] एक महत्तम पूर्णांक है जो x के समान अथवा उससे कम है, तो $\lim\limits_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$ बराबर है

(1) 1 (2) – sin 1 (3) 0 (4) sin 1

A. 2

Sol. $\lim\limits_{h\to 0} \frac{-h(-1+h)\sin(-1)}{h} = -\sin 1$

Question ID : 4165298855

Option 1 ID : 41652934881 **Option 2 ID : 41652934880**

Option 3 ID : 41652934879 Option 4 ID : 41652934878

10. If $\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta = 1 - \frac{1}{\sqrt{2}}, (k > 0)$, then the value of k is :

यदि $\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta = 1 - \frac{1}{\sqrt{2}}, (k > 0)$ है, तो k का मान है –

(1) 1 (2) 4 (3) 2 (4) $\frac{1}{2}$

A. 3

Sol. $\int_0^{\frac{\pi}{3}} \frac{\tan\theta\, d\theta}{\sqrt{2k\sec\theta}} = \int_0^{\frac{\pi}{3}} \frac{\tan\theta.\sec\theta\, d\theta}{\sqrt{2k}\sqrt{\sec\theta}\sec\theta}$

$\sec\theta = t$

$$\int_1^2 \frac{dt}{\sqrt{2k}\ t^{3/2}} = \frac{1}{\sqrt{2k}}\left(\frac{-2}{\sqrt{t}}\right)_1^2 = \frac{1}{\sqrt{2k}}\left(-\sqrt{2}+2\right) = 1 - \frac{1}{\sqrt{2}}$$

$\Rightarrow k = 2$

Question ID : 4165298860

Option 1 ID : 41652934899 | Option 2 ID : 41652934900

Option 3 ID : 41652934898 | Option 4 ID : 41652934901

11. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6i\ z_0^{81} - 3iz_0^{93}$, then arg z is equal to

माना कि द्विघातीय समीकरण $x^2 + x + 1 = 0$ का एक मूल z_0 है यदि $z = 3 + 6i\ z_0^{81} - 3iz_0^{93}$ है तो कोणांक z (arg z) बराबर है –

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) 0 (4) $\frac{\pi}{6}$

A. 2

Sol. $x^2 + x + 1 = 0$ has the roots w, w^2 w is imaginary cube root of unity

$z = 3 + 6i\ w^{81} - 3i\ w^{93}$

$z = 3 + 6i - 3i = 3 + 3i$

$$\arg(z) = \tan^{-1}\left(\frac{3}{3}\right) = \left(\frac{\pi}{4}\right)$$

Question ID : 4165298847

Option 1 ID : 41652934848 | **Option 2 ID : 41652934846**

Option 3 ID : 41652934849 | Option 4 ID : 41652934847

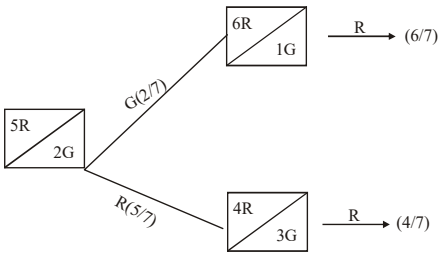
12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

एक कलश में 5 लाल तथा 2 हरी गेंदें है। इस कलश में से यादृच्छया एक गेंद निकली गई। यदि निकाली गई गेंद हरी है, तो कलश में एक लाल गेंद डाली जाती है तथा यदि निकाली गई गेंद लाल है, तो कलश में एक हरी गेंद डाली जाती है, जबकि निकाली गई गेंद वापिस नहीं डाली जाती । अब इसमें से यादृच्छया एक दूसरी गेंद निकाली गई, तो इस दूसरी गेंद के लाल होने की प्रायिकता है –

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

A. 2

Sol.



$$P(E) = \frac{2}{7} \times \frac{6}{7} + \frac{5}{7} \times \frac{4}{7}$$

$$P = \frac{32}{49}$$

Question ID : 4165298872

Option 1 ID : 41652934948 | **Option 2 ID : 41652934949**

Option 3 ID : 41652934946 Option 4 ID : 41652934947

13. The area of the region $A = \{(x, y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1\}$ in sq. units, is :

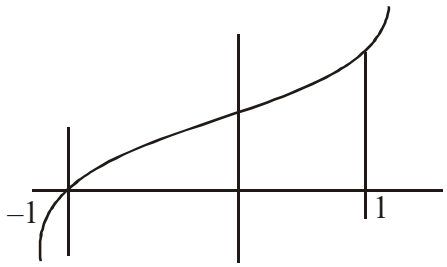
क्षेत्र $A = \{(x, y) : 0 \le y \le x|x| + 1$ तथा $-1 \le x \le 1\}$ का वर्ग इकाईयों में क्षेत्रफल है –

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) 2

A. 4

Sol. $A = \{(x, y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1\}$

for $0 \le x \le 1 \Rightarrow 0 \le y \le x^2 + 1$



$-1 \le x < 0 \Rightarrow 0 \le y \le -x^2 + 1$

$$A = \int_{-1}^{0} \left(-x^2 + 1\right) dx = \int_{0}^{1} \left(x^2 + 1\right) dx$$

$$A = \frac{2}{3} + \frac{4}{3} = 2$$

Question ID : 4165298861

Option 1 ID : 41652934903 Option 2 ID : 41652934902

Option 3 ID : 41652934904 **Option 4 ID : 41652934905**

14. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x - 4)^2 + (y - 7)^2 = 36$ intersect at two distinct point, then :

यदि वृत्त $x^2 + y^2 - 16x - 20y + 164 = r^2$ तथा $(x - 4)^2 + (y - 7)^2 = 36$ दो भिन्न बिन्दुओं पर काटते है, तो

(1) $r > 11$ (2) $r = 11$ (3) $1 < r < 11$ (4) $0 < r < 1$

A. 3

Sol. $(x - 8)^2 + (y - 10)^2 = r^2$

$(x - 4)^2 + (y - 7)^2 = 36$

$|r_1 - r_2| < c_1c_2 < r_1 + r_2$

$|r - 6| < 5 < r + 6$

$1 < r < 11$

Question ID : 4165298865

Option 1 ID : 41652934921 Option 2 ID : 41652934920

Option 3 ID : 41652934918 Option 4 ID : 41652934919

15. Let $f : [0, 1] \to R$ be such that $f(xy) = f(x).f(y)$, for all $x, y \in [0, 1]$, and $f(0) \ne 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is equal to :

माना $f : [0, 1] \to R$ इस प्रकार है कि सभी $x, y \in [0, 1]$ के लिए $f(xy) = f(x).f(y)$ है तथा $f(0) \ne 0$ है। यदि $y = y(x)$

अवकल समीकरण $\frac{dy}{dx} = f(x)$ को सन्तुष्ट करता है और $y(0) = 1$ है तो $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ बराबर है

(1) 5 (2) 3 (3) 2 (4) 4

A. 2

Sol. $f(xy) = f(x) \cdot f(y)$

$f(0) = 1$ as $f(0) \neq 0$

$\Rightarrow f(x) = 1, \frac{dy}{dx} = 1$

$\Rightarrow y = x + c \Rightarrow c = 1$

$\Rightarrow y = x + 1$

$\Rightarrow y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4} + 1 + \frac{3}{4} + 1 = 3$

Question ID : 4165298862

Option 1 ID : 41652934909 **Option 2 ID : 41652934907**

Option 3 ID : 41652934906 Option 4 ID : 41652934908

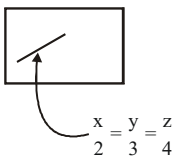
16. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is :

एक ऐसे समतल का समीकरण, जिस पर रेखा $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ स्थित है तथा जो एक अन्य समतल जिसमें रेखायें $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ तथा $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ स्थित है, के लंबवत है, है –

(1) $x + 2y - 2z = 0$ (2) $x - 2y + z = 0$ (3) $3x + 2y - 3z = 0$ (4) $5x + 2y - 4z = 0$

A. 2

Sol. $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}, \frac{x}{4} = \frac{y}{2} = \frac{z}{3}$



$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = \hat{i}(8) - \hat{j} - 10\hat{k}$ Perpendicular vector of both lines

normal vector of required plane $a\hat{i} + b\hat{j} + c\hat{k}$

$\left.\begin{matrix} 8a - b - 10c = 0 \\ 2a + 3b + 4c = 0 \end{matrix}\right\} \Rightarrow \frac{a}{26} = \frac{-b}{52} = \frac{c}{26}$

equation of plane $(x) - 2y + z = 0$

Question ID : 4165298869

Option 1 ID : 41652934934 **Option 2 ID : 41652934936**

Option 3 ID : 41652934935 Option 4 ID : 41652934937

17. If the system of linear equations

$x - 4y+7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then :

यदि रैखिक समीकरण निकाय

$x - 4y+7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

संगत (consistent) है तो

(1) $g + h + 2k = 0$ (2) $2g + h + k = 0$ (3) $g + h + k = 0$ (4) $g + 2h + k = 0$

A. 2

Sol. $\Delta = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 3 & -5 \\ -2 & 5 & -9 \end{vmatrix} = 0$

$\Delta_x = \begin{vmatrix} g & -4 & 7 \\ h & 3 & -5 \\ k & 5 & -9 \end{vmatrix} = -2g - h - k = 0 \Rightarrow 2g + h + k = 0$

Question ID : 4165298850

Option 1 ID : 41652934860 **Option 2 ID : 41652934858**

Option 3 ID : 41652934861 Option 4 ID : 41652934859

18. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to :

माना $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ तीन ऐसे सदिश है कि $\vec{b}$ का $\vec{a}$ पर प्रक्षेप सदिश $\vec{a}$ है। यदि $\vec{a} + \vec{b}$, सदिश $\vec{c}$ के लंबवत है, तो $|\vec{b}|$ बराबर है –

(1) 6 (2) $\sqrt{32}$ (3) $\sqrt{22}$ (4) 4

A. 1

Sol. $(\vec{a} + \vec{b}) \perp (\vec{c})$

$\Rightarrow (\vec{a} + \vec{b}).\vec{c} = 0 \Rightarrow (1 + b_1) + (1 + b_2) + 4 = 0 \Rightarrow 5b_1 + b_2 + 10 = 0$

Projection of $\vec{b}$ on $\vec{a} \Rightarrow \left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{a} = \vec{a}$

$\Rightarrow \vec{a}.\vec{b} = |\vec{a}|^2 \Rightarrow b_1 + b_2 + 2 = 4 \Rightarrow b_1 + b_2 = 2$

$b_1 = -3, b_2 = 5$

$\vec{b} = -3\hat{i} - 5\hat{j} + \sqrt{2}\hat{k}$

$|\vec{b}| = \sqrt{9+25+2} = 6$

Question ID : 4165298870

Option 1 ID : 41652934938 Option 2 ID : 41652934941

Option 3 ID : 41652934940 Option 4 ID : 41652934939

19. Let A (4, –4) and B (9, 6) be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔABC is maximum. Then, the area (in sq. units) of ΔACB, is :

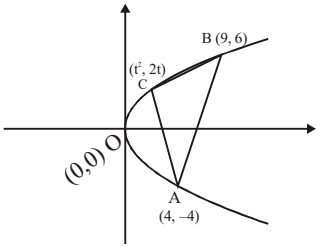
माना A (4, –4) तथा B (9, 6) एक परवलय $y^2 = 4x$ पर स्थित दो बिन्दु है। माना परवलय के चाप AOB (जहां O मूल बिन्दु है) पर स्थित एक बिन्दु C इस प्रकार चुना गया कि ΔABC का क्षेत्रफल अधिकतम है, तो ΔACB का क्षेत्रफल (वर्ग इकाईयों में) है :

(1) $30\frac{1}{2}$ (2) $31\frac{1}{4}$ (3) $31\frac{3}{4}$ (4) 32

A. 2

Sol. $A = \frac{1}{2}\begin{vmatrix} 4^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix}$

$A = 10\,(t^2 - t - 6)$



$\frac{dA}{dt} = 0 \Rightarrow (2t-1) = 0$

$t = \frac{1}{2}\left|10 \times \frac{1}{4} - \frac{10}{2} - 60\right| = \frac{125}{4}$

for $t = \frac{1}{2}, \frac{d^2A}{dt^2} < 0$

Question ID : 4165298866

Option 1 ID : 41652934923 **Option 2 ID : 41652934922**

Option 3 ID : 41652934925 Option 4 ID : 41652934924

20. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1,1), then the equation of its third side is :

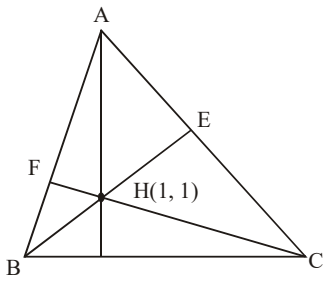
माना एक त्रिभुज की दो भुजाओं के समीकरण $3x - 2y + 6 = 0$ तथा $4x + 5y - 20 = 0$ है। यदि इस त्रिभुज का लंबकेन्द्र (1,1) पर है, तो इसकी तीसरी भुजा का समीकरण है –

(1) $26x - 122y - 1675 = 0$ (2) $122y + 26x + 1675 = 0$

(3) $122y - 26x - 1675 = 0$ (4) $26x + 61y + 1675 = 0$

A. 1

Sol.



Equation of AB $\rightarrow 3x - 2y + 6 = 0$

Equation of AC $\rightarrow 4x + 5y - 20 = 0$

Equation of BE $\rightarrow 2x + 3y - 5 = 0$

Equation of CF $\rightarrow 5x - 4y - 1 = 0$

find B and C

$\Rightarrow$ Equation of BC is $26x - 122y = 1675$

Question ID : 4165298864

Option 1 ID : 41652934915 Option 2 ID : 41652934917

Option 3 ID : 41652934914 Option 4 ID : 41652934916

21. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is :

$\left(\frac{1-t^6}{1-t}\right)^3$ के प्रसार में t^4 का गुणांक है –

(1) 15 (2) 14 (3) 12 (4) 10

A. 1

Sol. $(1 - t^6)^3 (1 - t)^{-3}$

$= (1 - t^{18} - 3t^6 + 3t^{12}) (1 - t)^{-3}$

coefficient of t^4 in $(1 - t)^{-3}$

$^{3+4-1}C_4 = {}^6C_2 = 15$

Question ID : 4165298852

Option 1 ID : 41652934868 Option 2 ID : 41652934869

Option 3 ID : 41652934866 Option 4 ID : 41652934867

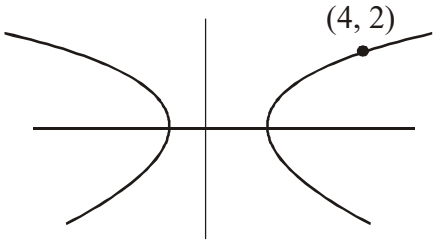
22. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

एक अतिपरवलय का केन्द्र मूल बिन्दु पर है तथा यह बिन्दु (4,2) से होकर जाता है और इसका अनुप्रस्थ (transverse) अक्ष, x-अक्ष के अनुदिश है जिसकी लम्बाई 4 है तो इस अतिपरवलय की उत्केन्द्रता (eccentricity) है –

(1) $\frac{3}{2}$ (2) $\sqrt{3}$ (3) 2 (4) $\frac{2}{\sqrt{3}}$

A. 4

Sol.



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$2a = 4 \Rightarrow a = 2$

Passes through (4, 2)

$$\frac{x^2}{4^2}-\frac{y^2}{b^2}=1\Rightarrow 4-\frac{4}{b^2}=1\Rightarrow b^2=\frac{4}{3}$$

$$e=\frac{2}{\sqrt{3}}$$

Question ID : 4165298867

Option 1 ID : 41652934929 Option 2 ID : 41652934926

Option 3 ID : 41652934928 **Option 4 ID : 41652934927**

23. Let f be a differentiable function from R to R such that $|f(x)-f(y)|\le 2|x-y|^{\frac{3}{2}}$ for all x, $y \in R$. If f(0) = 1 then $\int_0^1 f^2(x)dx$ is equal to :

माना f : R $\rightarrow$ R एक ऐसा अवकलनीय फलन है, कि सभी x, $y \in R$ के लिए $|f(x)-f(y)|\le 2|x-y|^{\frac{3}{2}}$ है। यदि f(0) = 1 है तो $\int_0^1 f^2(x)dx$ बराबर है –

(1) 1 (2) 0 (3) $\frac{1}{2}$ (4) 2

A. 1

Sol. $\left|\frac{f(x)-f(y)}{x-y}\right|\le 2\,|x-y|^{\frac{1}{2}}$

When $x \rightarrow y$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0 \Rightarrow f(y) = c$

$\Rightarrow f(x) = 1$

$$\int_0^1 f^2(x)dx=\int_0^1 1.dx=1$$

Question ID : 4165298858

Option 1 ID : 41652934891 Option 2 ID : 41652934890

Option 3 ID : 41652934892 Option 4 ID : 41652934893

24. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1,5], then m lies in the interval :

यदि द्विघात समीकरण $x^2 - mx + 4 = 0$ के दोनों मूल वास्तविक तथा भिन्न है और वे अन्तराल $[1,5]$ में स्थित है, तो m जिस अन्तराल में स्थित है, वह है –

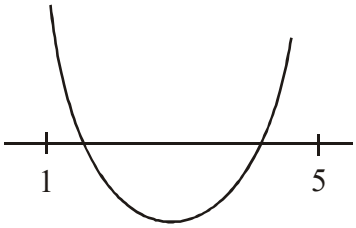
(1) (3,4) (2) (5,6) (3) (–5,–4) (4) (4,5)

A. 4

Sol. $x^2 - mx + 4 = 0, D > 0 \Rightarrow m^2 - 16 > 0 \Rightarrow m > 4, m < -4$

$f(1) > 0 \Rightarrow 5 - m > 0 \Rightarrow m < 5$

$f(5) > 0 \Rightarrow 29 - 5m > 0 \Rightarrow m < \frac{29}{5}$



$1 < \frac{b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow 2 < m < 10$

So $m \in (4, 5)$

Question ID : 4165298857

Option 1 ID : 41652934887 Option 2 ID : 41652934889

Option 3 ID : 41652934886 **Option 4 ID : 41652934888**

25. If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$ then A is :

(1) invertible only if $t = \pi$ (2) not invertible for any $t \in R$

(3) invertible for all $t \in R$ (4) invertible only if $t = \frac{\pi}{2}$

यदि $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$ है तो A :

(1) व्युत्क्रमणीय है, केवल $t = \pi$ है। (2) किसी भी $t \in R$ के लिए व्युत्क्रमणीय नहीं है।

(3) सभी $t \in R$ के लिए व्युत्क्रमणीय है। (4) व्युत्क्रमणीय है, केवल तब, जब $t = \frac{\pi}{2}$ है।

A. 3

Sol. $|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$

$$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1 = \begin{vmatrix} 1 & \cos t & \sin t \\ 0 & -\cos t - \sin t & -2\sin t + \cos t \\ 0 & 2\sin t - \cos t & -2\cos t - \sin t \end{vmatrix}$$

$= (2\cos t + \sin t)^2 + (2\sin t - \cos t)^2$

$|A| = 5$

So A is invertible for all $t \in R$

Question ID : 4165298849

Option 1 ID : 41652934854 Option 2 ID : 41652934857

Option 3 ID : 41652934856 Option 4 ID : 41652934855

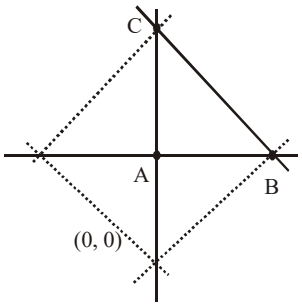
26. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is :

माना S, xy-तल में स्थित ऐसी सभी त्रिभुजों का समुच्चय है जिनका एक शीर्ष मूल बिन्दु पर है तथा दूसरे दो शीर्ष निर्देशांक अक्षों पर है तथा जिनके निर्देशांक पूर्णांकीय है। यदि S की प्रत्येक त्रिभुज का क्षेत्रफल 50 वर्ग इकाई है, तो समुच्चय S के अवयवों की संख्या है –

(1) 18 (2) 36 (3) 32 (4) 9

A. 2

Sol.



$$A = \frac{1}{2}|ab| = 50 \Rightarrow ab = 100$$

Prime factorisation of $100 = 2^2 \times 5^2$

total divisiors $= 3 \times 3 = 9$

So total elements in $S = 9 \times 4 = 36$

Question ID : 4165298863

Option 1 ID : 41652934911 **Option 2 ID : 41652934912**

Option 3 ID : 41652934913 Option 4 ID : 41652934910

27. If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$ is :

यदि $x = 3\tan t$ तथा $y = 3\sec t$ है, तो $t = \frac{\pi}{4}$ पर $\frac{d^2y}{dx^2}$ का मान है –

(1) $\frac{1}{6\sqrt{2}}$ (2) $\frac{1}{6}$ (3) $\frac{3}{2\sqrt{2}}$ (4) $\frac{1}{3\sqrt{2}}$

A. 1

Sol. $y^2 - x^2 = 9 \Rightarrow 2y\frac{dy}{dx} - 2x = 0, \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{3\times1}{3\times\sqrt{2}}$

$$\Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - 1 = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1-\left(\frac{dy}{dx}\right)^2_{t=\frac{\pi}{4}}}{(y)_{t=\frac{\pi}{4}}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1-\frac{1}{2}}{3\left(\sqrt{2}\right)} = \frac{1}{3.2\sqrt{2}} = \frac{1}{6\sqrt{2}}$$

Question ID : 4165298856

Option 1 ID : 41652934883 Option 2 ID : 41652934882

Option 3 ID : 41652934885 Option 4 ID : 41652934884

28. The sum of the following series $1+6+\frac{9\left(1^2+2^2+3^2\right)}{7}+\frac{12\left(1^2+2^2+3^2+4^2\right)}{9}+\frac{15\left(1^2+2^2+....+5^2\right)}{11}+....$ up to 15 terms, is :

निम्न श्रेणी $1+6+\frac{9\left(1^2+2^2+3^2\right)}{7}+\frac{12\left(1^2+2^2+3^2+4^2\right)}{9}+\frac{15\left(1^2+2^2+....+5^2\right)}{11}+....$ के प्रथम 15 पदों का योग है –

(1) 7510 (2) 7830 (3) 7820 (4) 7520

A. 3

Sol. $T_n = \frac{3n\left(1^2+2^2+....+n^2\right)}{(2n+1)} = \frac{3n.n(n+1)(2n+1)}{6(2n+1)}$

$$T_n = \frac{1}{2}\left(n^3+n^2\right)$$

$$S_{15} = \sum^{15} T_n = \frac{1}{2}\sum^{15} n^3 + \frac{1}{2}\sum^{15} n^2$$

$$= \frac{1}{2}\times\left(\frac{15\times16}{2}\right)^2 + \frac{1}{2}\times\frac{15\times16\times31}{6}$$

$= 7820$

Question ID : 4165298854

Option 1 ID : 41652934874 Option 2 ID : 41652934876

Option 3 ID : 41652934875 Option 4 ID : 41652934877

29. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which sin x – sin 2x + sin 3x = 0, is :

यदि $0 \le x < \frac{\pi}{2}$ है तो x के उन मानों की संख्या जिनके लिए sin x – sin 2x + sin 3x = 0 है, है –

(1) 3 (2) 2 (3) 1 (4) 4

A. 2

Sol. $\sin x + \sin 3x - \sin 2x = 0$

$\Rightarrow 2\sin 2x.\cos x - \sin 2x = 0 \Rightarrow \sin 2x\,(2\cos x - 1) = 0$

$\Rightarrow \cos x = 1/2 \qquad \sin 2x = 0$

$\Rightarrow x = \frac{\pi}{3}$ in $\left[0, \frac{\pi}{2}\right)$ $\quad x = 0$ in $\left[0, \frac{\pi}{2}\right)$

Question ID : 4165298873

Option 1 ID : 41652934950 **Option 2 ID : 41652934952**

Option 3 ID : 41652934953 Option 4 ID : 41652934951

30. If $f(x) = \int \frac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)^2} dx, (x \ge 0)$ and f(0) = 0, then the value of f(1) is :

यदि $f(x) = \int \frac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)^2} dx, (x \ge 0)$ तथा f(0) = 0 है तो f(1) का मान है –

(1) $\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) $\frac{1}{4}$ (4) $-\frac{1}{2}$

A. 3

Sol. $f(x) = \int \frac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)^2} dx = \int \frac{5x^8 + 7x^6}{x^{14}\left(\frac{1}{x^5} + \frac{1}{x^7} + 2\right)^2}$

$\frac{1}{x^5} + \frac{1}{x^7} + 2 = t$

$= \int \frac{-dt}{t^2} = \frac{1}{t} = \frac{1}{2 + \frac{1}{x^5} + \frac{1}{x^7}} + C$

As f(0) = 0 So $f(x) = \frac{x^7}{2x^7 + x^2 + 1}$

$f(1) = \frac{1}{4}$

Question ID : 4165298859

Option 1 ID : 41652934896 Option 2 ID : 41652934897

Option 3 ID : 41652934895 Option 4 ID : 41652934894