JEE Main January 2025
Question Paper With Text Solution
22 January | Shift-1

MATHEMATICS

JEE MAIN JANUARY 2025 | 22TH JANUARY SHIFT-1

SECTION – A

Question ID : 65644595

1. Let $x = x(y)$ be the solution of the differential equation $y^2\, dx + \left(x - \dfrac{1}{y}\right) dy = 0$. If $x(1) = 1$, then $x\left(\dfrac{1}{2}\right)$ is :

(1) $3 - e$ (2) $\dfrac{1}{2} + e$ (3) $\dfrac{3}{2} + e$ (4) $3 + e$

Ans. Official answer NTA (1)

Sol.

Question ID : 65644577

2. The number of non-empty equivalence relations on the set $\{1,2,3\}$ is :

(1) 7   (2) 6   (3) 4   (4) 5

Ans. Official answer NTA (4)

Sol.

Question ID : 65644582

3. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is ' M ', is :

(1) 14950   (2) 5148   (3) 4356   (4) 6084

Ans. Official answer NTA (2)

Sol.

Question ID : 65644579

4. Let z_1, z_2 and z_3 be three complex numbers on the circle $|z| = 1$ with $\arg\left(z_1\right) = \dfrac{-\pi}{4}$, $\arg(z_2) = 0$ and $\arg\left(z_3\right) = \dfrac{\pi}{4}$. If $\left|z_1\overline{z}_2 + z_2\overline{z}_3 + z_3\overline{z}_1\right|^2 = \alpha + \beta\sqrt{2}, \alpha, \beta \in Z$, then the value of $\alpha^2 + \beta^2$ is :

(1) 31   (2) 24   (3) 29   (4) 41

Ans. Official answer NTA (3)

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Sol.

Question ID : 65644587

5. Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points P, Q and R. If the circle C with centre at $(-1, -1)$ passes through the points P, Q and R, then the area of ΔPQR is :

    (1) 4   (2) 7   (3) 6   (4) 5

Ans.   Official answer NTA (3)

Sol.

Question ID : 65644584

6. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\dfrac{m}{n}$, where gcd(m, n)=1, then m + n is equal to :

    (1) 4   (2) 14   (3) 13   (4) 11

Ans.   Official answer NTA (2)

Sol.

Question ID : 65644592

7. Let f(x) be a real differentiable function such that $f(0) = 1$ and $f(x + y) = f(x)\,f'(y) + f'(x)\,f(y)$ for all $x, y \in R$. Then $\displaystyle\sum_{n=1}^{100} \log_e f(n)$ is equal to :

    (1) 2406   (2) 5220   (3) 2525   (4) 2384

Ans.   Official answer NTA (3)

Sol.

Question ID : 65644589

8. Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16((\sec^{-1}x)^2 + (\operatorname{cosec}^{-1}x)^2)$ is :

    (1) $31\pi^2$ (2) $24\pi^2$ (3) $22\pi^2$ (4) $18\pi^2$

Ans.   Official answer NTA (3)

Sol.

Question ID : 65644580

9.   If $\sum_{r=1}^{n} T_r = \dfrac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$,then $\lim_{n \to \infty} \sum_{r=1}^{n} \left(\dfrac{1}{T_r} \right)$ is equal to :

(1) $\dfrac{1}{3}$ (2) 1   (3) $\dfrac{2}{3}$ (4) 0

Ans.   Official answer NTA (3)

Sol.

Question ID : 65644594

10.   The area of the region, inside the circle $\left(x - 2\sqrt{3} \right)^2 + y^2 = 12$ and outside the parabola $y^2 = 2\sqrt{3}x$ is :

(1) $3\pi + 8$ (2) $6\pi - 16$ (3) $3\pi - 8$ (4) $6\pi - 8$

Ans.

Ans.   Official answer NTA (2)

Question ID : 65644576

11.   Let A = {1, 2, 3, ..., 10} and $B = \left\{ \dfrac{m}{n} : m, n \in A, m < n \text{ and } \gcd(m, n) = 1 \right\}$. Then n(B) is equal to :

(1) 37   (2) 36   (3) 29   (4) 31

Ans.   Official answer NTA (4)

Sol.

Question ID : 65644578

12.   The product of all solutions of the equation $e^{5(\log_e x)^2 + 3} = x^8, x > 0$, is :

(1) $e^{\frac{6}{5}}$ (2) $e^{\frac{8}{5}}$ (3) e   (4) e^2

Ans.   Official answer NTA (2)

Sol.

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Question ID : 65644581

13. Let $a_1, a_2, a_3, \ldots\ldots$ be a G.P. of increasing positive terms. if $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, then a_6 is :

(1) 628　　　　　(2) 526　　　　　(3) 784　　　　　(4) 812

Ans.　Official answer NTA (3)

Sol.

Question ID : 65644585

14. Let the triangle PQR be the image of the triangle with vertices $(1, 3), (3, 1)$ and $(2, 4)$ in the line $x + 2y = 2$. If the centroid of ΔPQR is the point (α, β), then $15(\alpha - \beta)$ is equal to :

(1) 19　　　　　(2) 21　　　　　(3) 24　　　　　(4) 22

Ans.　Official answer NTA (4)

Sol.

Question ID : 65644586

15. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point $(2, 5)$ and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to :

(1) 10　　　　　(2) 15　　　　　(3) 14　　　　　(4) 12

Ans.　Official answer NTA (2)

Sol.

Question ID : 65644590

16. Let $L_1 : \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $L_2 : \dfrac{x-2}{3} = \dfrac{y-4}{4} = \dfrac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 :

(1) $\left(-\dfrac{5}{3}, -7, 1\right)$　　　(2) $\left(\dfrac{8}{3}, -1, \dfrac{1}{3}\right)$　　　(3) $\left(2, 3, \dfrac{1}{3}\right)$　　　(4) $\left(\dfrac{14}{3}, -3, \dfrac{22}{3}\right)$

Ans.　Official answer NTA (4)

Sol.

Question ID : 65644583

17.   A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64(\mu + \sigma^2)$ is :

(1) 64   (2) 48   (3) 32   (4) 51

Ans.   Official answer NTA (2)

Sol.

Question ID : 65644591

18.   Let $f : R \rightarrow R$ be a twice differentiable function such that $f(x + y) = f(x) f(y)$ for all $x, y \in R$. If $f'(0) = 4a$ and f satisfies $f''(x) - 3af'(x) - f(x) = 0, a > 0$, then the area of the region $R = \{(x, y) \mid 0 \le y \le f(ax), 0 \le x \le 2\}$ is:

(1) $e^2 + 1$ (2) $e^4 + 1$ (3) $e^2 - 1$ (4) $e^4 - 1$

Ans.   Official answer NTA (3)

Sol.

Question ID : 65644593

19.   Let for $f(x) = 7 \tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$, $I_1 = \int_0^{\pi/4} f(x)dx$ and $I_2 = \int_0^{\pi/4} x\, f(x)dx$. then $7I_1 + 12I_2$ is equal to :

(1) 2   (2) 2π (3) π (4) 1

Ans.   Official answer NTA (4)

Sol.

Question ID : 65644588

20.   Let the foci of a hyperbola be (1, 14) and (1, –12). If it passes through the point (1, 6), then the length of its latus-rectum is :

(1) $\dfrac{288}{5}$ (2) $\dfrac{144}{5}$ (3) $\dfrac{24}{5}$ (4) $\dfrac{25}{6}$

Ans.   Official answer NTA (1)

Sol.

SECTION – B

Question ID : 65644597

21. If $\displaystyle\sum_{r=0}^{5} \frac{^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$, gcd(m, n) = 1, then m – n is equal to _____ .

Ans. Official answer NTA (2035)

Sol.

Question ID : 65644599

22. Let $L_1 : \dfrac{x-1}{3} = \dfrac{y-1}{-1} = \dfrac{z+1}{0}$ and $L_2 : \dfrac{x-2}{2} = \dfrac{y}{0} = \dfrac{z+4}{\alpha}$, $a \in R$ be two lines, which intersect at the point B.

If P is the foot of perpendicular from the point A(1, 1, –1) on L_2, then the value of 26 α(PB)^2 is _____ .

Ans. Official answer NTA (216)

Sol.

Question ID : 656445100

23. Let the function,

$$f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ a^2 + bx, & x \geq 1 \end{cases}$$

be differentiable for all $x \in R$, where a > 1, b $\in$ R. If the area of the region enclosed by y = f(x) and the line

y = –20 is $\alpha + \beta\sqrt{3}, \alpha, \beta \in Z$, then the value of $\alpha + \beta$ is _____ .

Ans. Official answer NTA (34)

Sol.

Question ID : 65644596

24. Let A be a square matrix of order 3 such that det(A) = –2 and det(3adj (–6adj (3A))) = 2^{m+n} . 3^{mn}, m> n. Then

4m + 2n is equal to _____ .

Ans. Official answer NTA (34)

Sol.

Question ID : 65644598

25. Let $\vec{c}$ be the projection vector $\vec{b} = \lambda\hat{i} + 4\hat{k}$, $\lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{a}$ is _____.

Ans. Official answer NTA (16)

Sol.