JEE Main January 2025

Question Paper With Text Solution

22 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2025 | 22TH JANUARY SHIFT-2

SECTION – A

Question ID : 656445238

1. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a}+2\vec{b}$ and $3\vec{a}-\lambda\vec{b}$ are perpendicular to each other, then the number of values of λ in $[-1, 3]$ is :

(1) 1 (2) 2 (3) 3 (4) 0

Ans. Official answer NTA (4)

Sol.

Question ID : 656445237

2. The sum of all values of $\theta \in [0, 2\pi]$ satisfying $2\sin^2\theta = \cos 2\theta$ and $2\cos^2\theta = 3\sin\theta$ is :

(1) π (2) $\frac{5\pi}{6}$ (3) 4π (4) $\frac{\pi}{2}$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445228

3. Let the curve $z(1+i)+\bar{z}(1-i)=4, z \in C$, divide the region $|z-3| \le 1$ inito two parts of areas α and β. Then $|\alpha-\beta|$ is equal to :

(1) $1+\frac{\pi}{2}$ (2) $1+\frac{\pi}{6}$ (3) $1+\frac{\pi}{3}$ (4) $1+\frac{\pi}{4}$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445236

4. Let $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$ and $H:\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$. Let the distance between the foci of E and the foci of H be $2\sqrt{3}$. If $a-A=2$, and the ratio of the eccentricities of E and H is $\frac{1}{3}$, then the sum of the lengths of their latus rectums is equal to :

(1) 7 (2) 9 (3) 8 (4) 10

Ans. Official answer NTA (3)

Sol.

Question ID : 656445240

5. Let a line pass through two distinct points P(–2, –1, 3) and Q, and be parallel to the vector $3\hat{i}+2\hat{j}+2\hat{k}$. If the distance of the point Q from the point R(1, 3, 3) is 5, then the square of the area of ΔPQR is equal to :

(1) 148 (2) 140 (3) 136 (4) 144

Ans. Official answer NTA (3)

Sol.

Question ID : 656445235

6. Let $P\left(4,4\sqrt{3}\right)$ be a point on the parabola $y^2=4ax$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to :

(1) $\frac{34\sqrt{3}}{3}$ (2) $\frac{263\sqrt{3}}{8}$ (3) $\frac{343\sqrt{3}}{8}$ (4) $17\sqrt{3}$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445231

7. Suppose that the number of terms in an A.P. is 2k, $k \in N$. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to :

(1) 5 (2) 4 (3) 6 (4) 8

Ans. Official answer NTA (1)

Sol.

Question ID : 656445229

8. If the system of linear equations :

$x + y + 2z = 6$,
$2x + 3y + az = a + 1$,
$-x - 3y + bz = 2b$,

where $a, b \in R$, has infinitely many solutions, then $7a + 3b$ is equal to :

(1) 22 (2) 16 (3) 12 (4) 9

Ans. Official answer NTA (2)

Sol.

Question ID : 656445234

9. If A and B are two events such that $P(A \cap B) = 0.1$, and P(A|B) and P(B|A) are the roots of the equation $12x^2 - 7x + 1 = 0$, then the value of $\frac{P(\bar{A} \cup \bar{B})}{P(\bar{A} \cap \bar{B})}$ is :

(1) $\frac{9}{4}$ (2) $\frac{5}{3}$ (3) $\frac{7}{4}$ (4) $\frac{4}{3}$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445226

10. Let $A=\{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16\}$. Then the number of many-one functions $f: A \to B$ such that $1 \in f(A)$ is equal to :

(1) 139 (2) 151 (3) 163 (4) 127

Ans. Official answer NTA (2)

Ans.

Question ID : 656445239

11. The perpendicular distance, of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point P(2, –10, 1) is :

(1) $4\sqrt{3}$ (2) 6 (3) $5\sqrt{2}$ (4) $3\sqrt{5}$

Ans. Official answer NTA (4)

Sol.

Question ID : 656445241

12. If $\lim\limits_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)^{x}=\alpha$, then the value of $\frac{\log_e \alpha}{1+\log_e \alpha}$ equal to :

(1) e (2) e^{-2} (3) e^{2} (4) e^{-1}

Ans. Official answer NTA (1)

Sol.

Question ID : 656445227

13. Let α_θ and β_θ be the distinct roots of $2x^2+(\cos\theta)x-1=0, \theta\in(0,2\pi)$. if m and M are the minimum and the maximum values of $\alpha_\theta^4+\beta_\theta^4$, then $16(M+m)$ equals :

(1) 17 (2) 24 (3) 27 (4) 25

Ans. Official answer NTA (4)

Sol.

Question ID : 656445245

14. If $\int e^{x}\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\frac{\sin^{-1}x}{(1-x^2)^{3/2}}+\frac{x}{1-x^2}\right)dx=g(x)+C$, where C is the constant of integration, then $g\left(\frac{1}{2}\right)$ equals :

(1) $\frac{\pi}{6}\sqrt{\frac{e}{2}}$ (2) $\frac{\pi}{6}\sqrt{\frac{e}{3}}$ (3) $\frac{\pi}{4}\sqrt{\frac{e}{2}}$ (4) $\frac{\pi}{4}\sqrt{\frac{e}{3}}$

Ans. Official answer NTA (2)

Sol.

Question ID : 656445244

15. If x = f(y) is the solution of the differential equation $\left(1+y^2\right)+\left(x-2e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$, $y\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ with $f(0)=1$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(1) $e^{\frac{\pi}{6}}$ (2) $e^{\frac{\pi}{4}}$ (3) $e^{\frac{\pi}{12}}$ (4) $e^{\frac{\pi}{3}}$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445243

16. The area of the region enclosed by the curves $y=x^2-4x+4$ and $y^2=16-8x$ is :

(1) $\frac{4}{3}$ (2) 8 (3) 5 (4) $\frac{8}{3}$

Ans. Official answer NTA (4)

Sol.

Question ID : 656445242

17. Let $f(x)=\int_0^{x^2}\frac{t^2-8t+15}{e^t}dt, x\in R$ .Then the numbers of local maximum and local minimum"points of f, respectively, are :

(1) 2 and 3 (2) 1 and 3 (3) 3 and 2 (4) 2 and 2

Ans. Official answer NTA (1)

Sol.

Question ID : 656445230

18. For a 3×3 M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A|=\frac{1}{2}$ and trace (A) = 3. If B = adj(adj(2A)), then the value of |B| + trace (B) equals :

(1) 174 (2) 280 (3) 56 (4) 132

Ans. Official answer NTA (2)

Sol.

Question ID : 656445233

19. Let α, β, γ and δ be the coefficients of x^7, x^5, x^3 and x respectively in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $x>1$. If u and v satisfy the equations

$\alpha u + \beta v = 18$
$\gamma u + \delta v = 20$

then $u+v$ equals :

(1) 8 (2) 4 (3) 5 (4) 3

Ans. Official answer NTA (3)

Sol.

Question ID : 656445232

20. In a group of 3 girls and 4 boys, there are two boys B_1 and B_2. The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is :

(1) 144 (2) 120 (3) 72 (4) 96

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 656445246

21. If $\sum_{r=1}^{30}\frac{r^2\left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}}=\alpha\times 2^{29}$, then α is equal to _____.

Ans. Official answer NTA ()

Sol. 465

Question ID : 656445250

22. Let $A = \{1, 2, 3\}$. The number of relations on A, containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is _____.

Ans. Official answer NTA (3)

Sol.

Question ID : 656445249

23. Let the distance between two parallel lines be 5 units and a point P lie between the lines at a unit distance from one of them. An equilateral triangle PQR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to _____.

Ans. Official answer NTA (28)

Sol.

Question ID : 656445248

24. Let $y = f(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^6 + 4x}{\sqrt{1 - x^2}}, -1 < x < 1$ such that $f(0) = 0$.

If $6\int_{-1/2}^{1/2} f(x)dx = 2\pi - \alpha$ then α^2 is equal to _____.

Ans. Official answer NTA (27)

Sol.

Question ID : 656445247

25. Let A(6, 8), B(10 cos α, – 10 sin α) and C(–10 sin α, 10 cos α), be the vertices of a triangle. If L(a, 9) and G(h, k) be its orthocenter and centroid respectively, then (5a – 3h + 6k + 100 sin2α) is equal to _____.

Ans. Official answer NTA (145)

Sol.