JEE Main January 2025

Question Paper With Text Solution

23 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2025 | 23TH JANUARY SHIFT-1

SECTION – A

Question ID : 736475978

1. Let $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}, z \in C$, be the equation of a circle with center at C. If the area of the triangle,whose vertices are at the points (0,0), C and (α, 0) is 11 square units, then α^2 equals

(1) 50 (2) $\frac{121}{25}$ (3) 100 (4) $\frac{81}{25}$

Ans. Official answer NTA (3)

Sol.

Question ID : 736475986

2. The value of (sin 70°) (cot 10° cot 70° – 1) is :

(1) 2/3 (2) 3/2 (3) 1 (4) 0

Ans. Official answer NTA (3)

Sol.

Question ID : 736475979

3. If the system of equations

$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$

$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$

has infinitely many solutions, then $\lambda^2 + \lambda$ is equal to :

(1) 6 (2) 12 (3) 10 (4) 20

Ans. Official answer NTA (2)

Sol.

Question ID : 736475990

4. Let P be the foot of the perpendicular from the point Q(10,–3,–1) on the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$ Then the area of the right angled triangle PQR, where R is the point (3,–2,1), is :

(1) $3\sqrt{30}$ (2) $\sqrt{30}$ (3) $9\sqrt{15}$ (4) $8\sqrt{15}$

Ans. Official answer NTA (1)

Sol.

Question ID : 736475989

5. Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be $\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+3\hat{j}-2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is :

(1) $\frac{1}{6}\left(12\hat{i}+12\hat{j}+\hat{k}\right)$ (2) $\frac{1}{2}\left(\hat{i}+4\hat{j}+7\hat{k}\right)$ (3) $\frac{1}{6}\left(7\hat{i}+12\hat{j}+\hat{k}\right)$ (4) $\frac{1}{12}\left(7\hat{i}+4\hat{j}+3\hat{k}\right)$

Ans. Official answer NTA (3)

Sol.

Question ID : 736475980

6. If A, B, and $(\text{adj}(A^{-1})+\text{adj}(B^{-1}))$ are non-singular matrices of same order, then the inverse of $A(\text{adj}(A^{-1})+\text{adj}(B^{-1}))^{-1}B$, is equal to :

(1) $\text{adj}(B^{-1})+\text{adj}(A^{-1})$ (2) $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$

(3) $AB^{-1}+A^{-1}B$ (4) $\frac{1}{|AB|}(\text{adj}(B)+\text{adj}(A))$

Ans. Official answer NTA (4)

Sol.

Question ID : 736475977

7. Let R = (1,2), (2,3), (3,3) be a relation defined on the set {1,2,3,4}. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is :

(1) 10 (2) 7 (3) 9 (4) 8

Ans. Official answer NTA (2)

Sol.

Question ID : 736475991

8. If the function

$$f(x)=\begin{cases}\frac{2}{x}\{\sin(k_1+1)x+\sin(k_2-1)x\}, & x<0\\ 4, & x=0\\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right), & x>0\end{cases}$$

is continuous at x=0, then $k_1^2+k_2^2$ is equal to :

(1) 20 (2) 10 (3) 8 (4) 5

Ans. Official answer NTA (2)

Sol.

Question ID : 736475994

9. The value of $\int_{e^2}^{e^4}\frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}}+e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right)dx$ is :

(1) 2 (2) $\log_e 2$ (3) e^2 (4) 1

Ans. Official answer NTA (4)

Sol.

Question ID : 736475992

10. Marks obtain by all the students of class 12 are presented in a freqency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12 . If the number of students whose marks are less than 12 is 18 , then the total number of students is :

(1) 52 (2) 44 (3) 40 (4) 48

Ans.

Ans. Official answer NTA (2)

Question ID : 736475982

11. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is :

(1) 37000 (2) 36000 (3) 34000 (4) 35000

Ans. Official answer NTA (2)

Sol.

Question ID : 736475988

12. Let the arc AC of a circle subtend a right angle at the centre O. If the point B on the arc AC,divides the arc AC such that $\frac{\text{length of arc AB}}{\text{length of arc BC}}=\frac{1}{5}$, and $\overrightarrow{OC}=\alpha\overrightarrow{OA}+\beta\overrightarrow{OB}$, then $\alpha+\sqrt{2}\left(\sqrt{3}-1\right)\beta$ is equal to :

(1) $2+\sqrt{3}$ (2) $2-\sqrt{3}$ (3) $5\sqrt{3}$ (4) $2\sqrt{3}$

Ans. Official answer NTA (2)

Sol.

Question ID : 736475983

13. One die has two faces marked 1 , two faces marked 2 , one face marked 3 and one face marked 4 . Another die has one face marked 1, two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5 , when both the dice are thrown together, is :

(1) $\frac{4}{9}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{3}{5}$

Ans. Official answer NTA (3)

Sol.

Question ID : 736475981

14. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to :

(1) –1080 (2) –1200 (3) –120 (4) –1020

Ans. Official answer NTA (1)

Sol.

Question ID : 736475984

15. Let the area of a ΔPQR with vertices P(5,4), Q(–2,4) and R(a, b) be 35 square units. If its orthocenter and centroid are $O\left(2,\frac{14}{5}\right)$ and C(c, d) respectively, then c+2d is equal to :

(1) 2 (2) $\frac{8}{3}$ (3) $\frac{7}{3}$ (4) 3

Ans. Official answer NTA (4)

Sol.

Question ID : 736475995

16. Let a curve $y = f(x)$ pass through the points (0,5) and $(\log_e 2, k)$. If the curve satisfies the differential equation $2(3+y)e^{2x}\,dx - (7 + e^{2x})\,dy = 0$, then k is equal to :

(1) 16 (2) 8 (3) 32 (4) 4

Ans. Official answer NTA (2)

Sol.

Question ID : 736475976

17. Let $f(x) = \log_e x$ and $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$. Then the domain of fog is :

(1) R (2) $[1, \infty)$ (3) $[0, \infty)$ (4) $(0, \infty)$

Ans. Official answer NTA (1)

Sol.

Question ID : 736475993

18. Let $I(x)=\int\frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$. If $I(37)-I(24)=\frac{1}{4}\left(\frac{1}{b^{\frac{1}{13}}}-\frac{1}{c^{\frac{1}{13}}}\right)$, $b,c\in N$, then $3(b+c)$ is equal to :

(1) 40 (2) 39 (3) 26 (4) 22

Ans. Official answer NTA (2)

Sol.

Question ID : 736475985

19. If $\frac{\pi}{2}\le x\le\frac{3\pi}{4}$, then $\cos^{-1}\left(\frac{12}{13}\cos x+\frac{5}{13}\sin x\right)$ is equal to :

(1) $x-\tan^{-1}\frac{5}{12}$ (2) $x+\tan^{-1}\frac{4}{5}$ (3) $x+\tan^{-1}\frac{5}{12}$ (4) $x-\tan^{-1}\frac{4}{3}$

Ans. Official answer NTA (1)

Sol.

Question ID : 736475987

20. If the line $3x-2y+12=0$ intersects the parabola $4y=3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to :

(1) $\tan^{-1}\left(\frac{11}{9}\right)$ (2) $\tan^{-1}\left(\frac{4}{5}\right)$ (3) $\frac{\pi}{2}-\tan^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}\left(\frac{9}{7}\right)$

Ans. Official answer NTA (4)

Sol.

SECTION – B

Question ID : 736475997

21. If the area of the larger portion bounded between the curves $x^2+y^2=25$ and $y=|x-1|$ is $\frac{1}{4}(b\pi+c)$, $b,c\in N$, then $b+c$ is equal to _____.

Ans. Official answer NTA (77)

Sol.

Question ID : 7364751000

22. If the set of all values of a, for which the equation $5x^3 - 15x - a = 0$ has three distinct real roots, is the interval (α, β), then $\beta - 2\alpha$ is equal to _____.

Ans. Official answer NTA (30)

Sol.

Question ID : 736475996

23. If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, where $a + c = 15$ and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to _____.

Ans. Official answer NTA (117)

Sol.

Question ID : 736475998

24. The sum of all rational terms in the expansion of $\left(1 + 2^{\frac{1}{3}} + 3^{\frac{1}{2}}\right)^6$ is equal to _____.

Ans. Official answer NTA (612)

Sol.

Question ID : 736475999

25. Let the circle C touch the line $x - y + 1 = 0$, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x + 2y = 1$. Let H be the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2 + 3\beta^2$ is equal to _____.

Ans. Official answer NTA (19)

Sol.