JEE Main January 2025

Question Paper With Text Solution

23 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2025 | 23TH JANUARY SHIFT-2

SECTION – A

Question ID : 656445376

1. Let $A = \{(x, y) \in R \times R: |x + y| \geq 3\}$ and $B = \{(x, y) \in R \times R : |x| + |y| \leq 3\}$. If $C = \{(x, y) \in A \cap B : x = 0$ or $y = 0\}$, then $\sum_{(x,y)\in C} |x + y|$ is :

(1) 15 (2) 18 (3) 12 (4) 24

Ans. Official answer NTA (3)

Sol.

Question ID : 656445394

2. Let $x = x(y)$ be the solution of the differential equation $y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right)$, $y > 0$ and $x(1) = \frac{\pi}{2}$. Then $\cos(x(2))$ is equal to :

(1) $2(\log_e 2)^2 - 1$ (2) $2(\log_e 2) - 1$ (3) $1 - 2(\log_e 2)$ (4) $1 - 2(\log_e 2)^2$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445390

3. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm , the ice-cream melts at the rate of 81 cm^3/min and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi}$ cm / min . The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is :

(1) 196π (2) 225π (3) 128π (4) 256π

Ans. Official answer NTA (4)

Sol.

Question ID : 656445391

4. If the area of the region $\{(x, y) : -1 \le x \le 1, 0 \le y \le a + e^{|x|} - e^{-x}, a > 0\}$ is $\frac{e^2 + 8e + 1}{e}$, then the value of a is :

(1) 7 (2) 8 (3) 5 (4) 6

Ans. Official answer NTA (3)

Sol.

Question ID : 656445384

5. The length of the chord of the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$, whose mid-point is $\left(1, \frac{1}{2}\right)$, is :

(1) $\frac{5}{3}\sqrt{15}$ (2) $\frac{1}{3}\sqrt{15}$ (3) $\frac{2}{3}\sqrt{15}$ (4) $\sqrt{15}$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445378

6. The number of complex numbers z, satisfying $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$, is :

(1) 6 (2) 4 (3) 10 (4) 8

Ans. Official answer NTA (4)

Sol.

Question ID : 656445393

7. If $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$, then $\int_0^{21} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ equals :

(1) $\frac{\pi^2}{12}$ (2) $\frac{\pi^2}{4}$ (3) $\frac{\pi^2}{16}$ (4) $\frac{\pi^2}{8}$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445386

8. Let the range of the function $f(x)=6+16\cos x\cdot\cos\left(\frac{\pi}{3}-x\right)\cdot\cos\left(\frac{\pi}{3}+x\right)\cdot\sin 3x\cdot\cos 6x$, $x\in R$ be $[\alpha,\beta]$.

Then the distance of the point (α,β) from the line $3x+4y+12=0$ is :

(1) 8 (2) 11 (3) 9 (4) 10

Ans. Official answer NTA (2)

Sol.

Question ID : 656445388

9. If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$ and $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then m + n is equal to :

(1) 6 (2) 14 (3) 9 (4) 21

Ans. Official answer NTA (3)

Sol.

Question ID : 656445383

10. A rod of length eight units moves such that its ends A and B always lie on the lines $x-y+2=0$ and $y+2=0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio 2 : 1 is $9(x^2+\alpha y^2+\beta xy+\gamma x+28y)-76=0$, then $\alpha-\beta-\gamma$ is equal to :

(1) 22 (2) 24 (3) 21 (4) 23

Ans. Official answer NTA (4)

Sol.

Question ID : 656445389

11. $\lim\limits_{x\to\infty}\frac{\left(2x^2-3x+5\right)(3x-1)^{\frac{x}{2}}}{\left(3x^2+5x+4\right)\sqrt{(3x+2)^x}}$ is equal to :

(1) $\frac{2}{3\sqrt{e}}$ (2) $\frac{2e}{3}$ (3) $\frac{2}{\sqrt{3e}}$ (4) $\frac{2e}{\sqrt{3}}$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445380

12. Let $A = [a_{ij}]$ be a 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}, A\begin{bmatrix}4\\1\\3\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals :

(1) 2 (2) 0 (3) 1 (4) –1

Ans. Official answer NTA (4)

Sol.

Question ID : 656445377

13. Let $X = R \times R$. Define a relation R on X as :

$(a_1, b_1)\ R(a_2, b_2) \Leftrightarrow b_1 = b_2\}$.

Statement I: R is an equivalence relation.

Statement II : For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)\ R\ (a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Ans. Official answer NTA (1)

Sol.

Question ID : 656445379

14. The system of equations

$x + y + z = 6$,

$x + 2y + 5z = 9$,

$x + 5y + \lambda z = \mu$,

has no solution if :

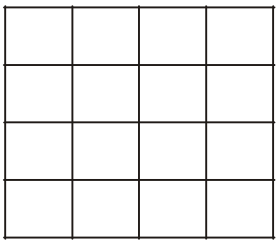
(1) $\lambda = 17, \mu \neq 18$ (2) $\lambda = 17, \mu = 18$ (3) $\lambda = 15, \mu \neq 17$ (4) $\lambda \neq 17, \mu \neq 18$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445382

15. A board has 16 squares as shown in the figure :



Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is :

(1) $\frac{4}{5}$ (2) $\frac{3}{5}$ (3) $\frac{23}{30}$ (4) $\frac{7}{10}$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445381

16. If in the expansion of $(1+x)^p(1-x)^q$, the coefficients of x and x^2 are 1 and -2, respectively, then p^2+q^2 is equal to :

(1) 18 (2) 8 (3) 13 (4) 20

Ans. Official answer NTA (3)

Sol.

Question ID : 656445385

17. Let the shortest distance from (a, 0), $a > 0$, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point (a, 0) and the focus of the parabola, and having its centre on the axis of the parabola is :

(1) $x^2 + y^2 - 10x + 9 = 0$ (2) $x^2 + y^2 - 8x + 7 = 0$

(3) $x^2 + y^2 - 6x + 5 = 0$ (4) $x^2 + y^2 - 4x + 3 = 0$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445387

18. The distance of the line $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ from the point (1, 4, 0) along the line $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ is :

(1) $\sqrt{13}$ (2) $\sqrt{17}$ (3) $\sqrt{14}$ (4) $\sqrt{15}$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445392

19. Let $\int x^3 \sin x dx = g(x) + C$, where C is the constant of integration. If $8\left(g\left(\frac{\pi}{2}\right)+g'\left(\frac{\pi}{2}\right)\right)=\alpha\pi^3+\beta\pi^2+\gamma$, $\alpha, \beta, \gamma \in Z$, then $\alpha + \beta - \gamma$ equals :

(1) 48 (2) 55 (3) 62 (4) 47

Ans. Official answer NTA (2)

Sol.

Question ID : 656445395

20. Let the point A divide the line segment joining the points P(–1, –1, 2) and Q(5, 5, 10) internally in the ratio r : 1(r > 0). If O is the origin and $\left(\overrightarrow{OQ}\cdot\overrightarrow{OA}\right)-\frac{1}{5}\left|\overrightarrow{OP}\times\overrightarrow{OA}\right|^2=10$, then the value of r is :

(1) 3 (2) 7 (3) $\sqrt{7}$ (4) 14

Ans. Official answer NTA (2)

Sol.

SECTION – B

Question ID : 656445396

21. The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2p$ is equal to _____.

Ans. Official answer NTA (474)

Sol.

Question ID : 656445400

22. The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5. If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and λ_2, $\lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _____.

Ans. Official answer NTA (15)

Sol.

Question ID : 656445397

23. Let α, β be the roots of the equation $x^2 - ax - b = 0$ with $\text{Im}(\alpha) < \text{Im}(\beta)$. Let $P_n = \alpha^n - \beta^n$. If $P_3 = -5\sqrt{7}i, P_4 = -3\sqrt{7}i, P_5 = 11\sqrt{7}i$ and $P_6 = 45\sqrt{7}i$, then $|\alpha^4 + \beta^4|$ is equal to _____.

Ans. Official answer NTA (31)

Sol.

Question ID : 656445398

24. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is _____.

Ans. Official answer NTA (17280)

Sol.

Question ID : 656445399

25. The variance of the numbers 8, 21, 34, 47, ..., 320 is _____.

Ans. Official answer NTA (8788)

Sol.