JEE Main January 2025

Question Paper With Text Solution

24 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2025 | 24TH JANUARY SHIFT-1

SECTION – A

Question ID : 7364751067

1. Consider the region $R = \left\{(x, y) : x \le y \le 9 - \frac{11}{3}x^2, x \ge 0\right\}$. The area, of the largest rectangle of sides parallel to the coordinate axes and inscribed in R, is :

(1) $\frac{821}{123}$ (2) $\frac{730}{119}$ (3) $\frac{625}{111}$ (4) $\frac{567}{121}$

Ans. Official answer NTA (4)

Sol.

Question ID : 7364751060

2. Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha, \beta)$, with $\beta < 4$, lies on C such that the length of the arc AB is $(1/6)^{th}$ of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to :

(1) $4 - \sqrt{3}$ (2) $3 + \sqrt{3}$ (3) 3 (4) 4

Ans. Official answer NTA (4)

Sol.

Question ID : 7364751064

3. Let in a ΔABC, the length of the side AC be 6, the vertex B be (1, 2, 3) and the vertices A, C lie on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Then the area (in sq. units) of ΔABC is :

(1) 17 (2) 21 (3) 56 (4) 42

Ans. Official answer NTA (2)

Sol.

Question ID : 7364751058

4. A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8, and B wins if he throws a sum of 8 before A throws a sum of 5 . The probability, that A wins if A makes the first throw, is:

(1) $\frac{9}{19}$ (2) $\frac{8}{19}$ (3) $\frac{8}{17}$ (4) $\frac{9}{17}$

Ans. Official answer NTA (1)

Sol.

Question ID : 7364751052

5. The product of all the rational roots of the equation $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 3$, is equal to :

(1) 7 (2) 14 (3) 21 (4) 28

Ans. Official answer NTA (2)

Sol.

Question ID : 7364751056

6. For some $n \neq 10$, let the coefficients of the 5^{th}, 6^{th} and 7^{th} terms in the binomial expansion of $(1 + x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1 + x)^{n+4}$ is :

(1) 70 (2) 20 (3) 35 (4) 10

Ans. Official answer NTA (3)

Sol.

Question ID : 7364751068

7. If $I(m, n) = \int_0^1 x^{m-1} (1 - x)^{n-1} dx$, $m, n > 0$, then $I(9, 14) + I(10, 13)$ is :

(1) I(9, 13) (2) I(1, 13) (3) I(19, 27) (4) I(9, 1)

Ans. Official answer NTA (1)

Sol.

Question ID : 7364751070

8. Let $y = y(x)$ be the solution of the differential equation $\left(xy - 5x^2\sqrt{1+x^2}\right)dx + \left(1+x^2\right)dy = 0, y(0) = 0$. Then $y\left(\sqrt{3}\right)$ is equal to :

(1) $2\sqrt{2}$ (2) $\sqrt{\frac{14}{3}}$ (3) $\frac{5\sqrt{3}}{2}$ (4) $\sqrt{\frac{15}{2}}$

Ans. Official answer NTA (3)

Sol.

Question ID : 7364751061

9. Let the product of the focal distances of the point $\left(\sqrt{3}, \frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is :

(1) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$ (2) $\frac{1-\sqrt{3}}{\sqrt{2}}$ (3) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$ (4) $\frac{1-2\sqrt{2}}{\sqrt{3}}$

Ans. Official answer NTA (3)

Sol.

Question ID : 7364751065

10. Let the line passing through the points (–1, 2, 1) and parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ intersect the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$ at the point P. Then the distance of P from the point Q(4, –5, 1) is :

(1) $5\sqrt{5}$ (2) 10 (3) 5 (4) $5\sqrt{6}$

Ans.

Ans. Official answer NTA (1)

Question ID : 7364751066

11. Let $f : R - \{0\} \rightarrow R$ be a function such that $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$. If the $\lim_{x\to 0}\left(\frac{1}{\alpha x} + f(x)\right) = \beta$; $\alpha, \beta \in R$, then $\alpha + 2\beta$ is equal to :

(1) 3 (2) 5 (3) 4 (4) 6

Ans. Official answer NTA (3)

Sol.

Question ID : 7364751063

12. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{c}$ is perpendicular to $\vec{b}$ and $\vec{a}\cdot\vec{c} = 5$, then $|\vec{c}|$ is equal to :

(1) 16 (2) 18 (3) $\sqrt{\frac{11}{6}}$ (4) $\frac{1}{3\sqrt{2}}$

Ans. Official answer NTA (3)

Sol.

Question ID : 7364751053

13. If α and β are the roots of the equation $2z^2 - 3z - 2i = 0$, where $i = \sqrt{-1}$, then

$16 \cdot \mathrm{Re}\left(\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}\right) \cdot \mathrm{Im}\left(\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}\right)$ is equal to :

(1) 398 (2) 409 (3) 441 (4) 312

Ans. Official answer NTA (3)

Sol.

Question ID : 7364751062

14. $\lim_{x\to 0} \operatorname{cosec} x\left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$ is :

(1) $-\frac{1}{2\sqrt{5}}$ (2) $\frac{1}{\sqrt{15}}$ (3) 0 (4) $\frac{1}{2\sqrt{5}}$

Ans. Official answer NTA (1)

Sol.

Question ID : 7364751054

15. If the system of equations

$2x - y + z = 4$
$5x + \lambda y + 3z = 12$
$100x - 47y + \mu z = 212$

has infinitely many solutions, then $\mu - 2\lambda$ is equal to :

(1) 59 (2) 57 (3) 55 (4) 56

Ans. Official answer NTA (2)

Sol.

Question ID : 7364751059

16. Let the lines $3x - 4y - \alpha = 0$, $8x - 11y - 33 = 0$, and $2x - 3y + \lambda = 0$ be concurrent. If the image of the point(1, 2) in the line $2x - 3y + \lambda = 0$ is $\left(\frac{57}{13}, \frac{-40}{13}\right)$, then $|\alpha\lambda|$ is equal to :

(1) 91 (2) 84 (3) 101 (4) 113

Ans. Official answer NTA (1)

Sol.

Question ID : 7364751055

17. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$ upto n terms. If the sum of the first six terms of an A.P. with first term $-p$ and common difference p is $\sqrt{2026 S_{2025}}$, then the absolute difference betwen 20^{th} and 15^{th} terms of the A.P. is :

(1) 25 (2) 90 (3) 20 (4) 45

Ans. Official answer NTA (1)

Sol.

Question ID : 7364751069

18. The area of region $\{(x, y) : x^2 + 4x + 2 \leq y \leq |x + 2|\}$ is equal to :

(1) 5 (2) 20/3 (3) 7 (4) 24/5

Ans. Official answer NTA (2)

Sol.

Question ID : 7364751057

19. For a statistical data $x_1, x_2, ..., x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$. He later found that he had noted two values in the data incorrectly as 4 and 5, instead of the correct values 6 and 8, respectively. The variance of the corrected data is :

(1) 9 (2) 4 (3) 7 (4) 5

Ans. Official answer NTA (3)

Sol.

Question ID : 7364751051

20. Let $f(x) = \frac{2^{x+2} + 16}{2^{2x+1} + 2^{x+4} + 32}$. Then the value of $8\left(f\left(\frac{1}{15}\right) + f\left(\frac{2}{15}\right) + \ldots + f\left(\frac{59}{15}\right)\right)$ is equal to :

(1) 108 (2) 102 (3) 118 (4) 92

Ans. Official answer NTA (3)

Sol.

SECTION – B

Question ID : 7364751073

21. The number of 3-digit numbers, that are divisible by 2 and 3 , but not divisible by 4 and 9 , is _________.

Ans. Official answer NTA ()

Sol. 125

Question ID : 7364751071

22. Let $S = \{p_1, p_2 ...., p_{10}\}$ be the set of first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct elements of S. Then the number of all ordered pairs (x, y), $x \in S$, $y \in A$, such that x divides y, is ________.

Ans. Official answer NTA (5120)

Sol.

Question ID : 7364751074

23. If for some α, β; $\alpha \le \beta$, $\alpha + \beta = 8$ and $\sec^2(\tan^{-1}\alpha) + \mathrm{cosec}^2(\cot^{-1}\beta) = 36$, then $\alpha^2 + \beta$ is ________.

Ans. Official answer NTA (14)

Sol.

Question ID : 7364751075

24. Let f be a differentiable function such that $2(x+2)^2 f(x) - 3(x+2)^2 = 10\int_0^x (t+2)f(t)dt$, $x \ge 0$. Then f(2) is equal to ________.

Ans. Official answer NTA (19)

Sol.

Question ID : 7364751072

25. Let A be a 3 × 3 matrix such that $X^T AX = O$ for all nonzero 3 × 1 matrices $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If

$A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, A\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and $\det(\mathrm{adj}(2(A+I))) = 2^\alpha 3^\beta 5^\gamma$, $\alpha, \beta, \gamma \in N$, then $\alpha^2 + \beta^2 + \gamma^2$ is ________.

Ans. Official answer NTA (44)

Sol.