JEE Main January 2025

Question Paper With Text Solution

24 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2025 | 24TH JANUARY SHIFT-2

SECTION – A

Question ID : 656445453

1. The number of real solution(s) of the equation $x^2 + 3x + 2 = \min\{|x-3|, |x+2|\}$ is :

(1) 2 (2) 0 (3) 3 (4) 1

Ans. Official answer NTA (1)

Sol.

Question ID : 656445452

2. Let $A = \left\{x \in (0,\pi) - \left\{\frac{\pi}{2}\right\} : \log_{(2/\pi)} |\sin x| + \log_{(2/\pi)} |\cos x| = 2\right\}$ and

$B = \left\{x \geq 0 : \sqrt{x}\left(\sqrt{x} - 4\right) - 3\left|\sqrt{x} - 2\right| + 6 = 0\right\}$. Then $n(A \cup B)$ is equal to :

(1) 4 (2) 2 (3) 6 (4) 8

Ans. Official answer NTA (4)

Sol.

Question ID : 656445467

3. Let [x] denote the greatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x) = [x] + |x-2|, -2 < x < 3$, is not continuous and not differentiable. Then $m + n$ is equal to : :

(1) 6 (2) 8 (3) 7 (4) 9

Ans. Official answer NTA (2)

Sol.

Question ID : 656445456

4. If $7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \frac{1}{7^3}(5+3\alpha) + \ldots\ldots\ldots\infty$, then the value of α is :

(1) 1 (2) $\frac{1}{7}$ (3) $\frac{6}{7}$ (4) 6

Ans. Official answer NTA (4)

Sol.

Question ID : 656445457

5. In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to :

(1) 525 (2) 515 (3) 510 (4) 505

Ans. Official answer NTA (2)

Sol.

Question ID : 656445470

6. Let $f : (0, \infty) \to R$ be a function which is differentiable at all points of its domain and satisfies the condition $x^2 f'(x) = 2xf(x) + 3$, with $f(1) = 4$. Then $2f(2)$ is equal to :

(1) 23 (2) 29 (3) 19 (4) 39

Ans. Official answer NTA (4)

Sol.

Question ID : 656445458

7. Let $A = [a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1 . Let E be the event that A is an invertible matrix. Then the probability P(E) is :

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$ (3) $\frac{3}{16}$ (4) $\frac{3}{8}$

Ans. Official answer NTA (4)

Sol.

Question ID : 656445461

8. Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides $x + y = 11$, $x + 2y = 16$ and $2x + 3y = 29$. Then the product of the smallest and the largest values of α is equal to : :

(1) 22 (2) 44 (3) 33 (4) 55

Ans. Official answer NTA (3)

Sol.

Question ID : 656445454

9. For some a, b, let $f(x)=\begin{vmatrix} a+\frac{\sin x}{x} & 1 & b \\ a & 1+\frac{\sin x}{x} & b \\ a & 1 & b+\frac{\sin x}{x} \end{vmatrix}$, $x \neq 0$, $\lim\limits_{x\to 0} f(x)=\lambda+\mu a+\nu b$. Then $(\lambda+\mu+\nu^2)$ is equal to :

(1) 9 (2) 36 (3) 16 (4) 25

Ans. Official answer NTA (3)

Sol.

Question ID : 656445466

10. Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}, -5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha+2\beta+5\gamma$ is equal to :

(1) 6 (2) 3 (3) 1 (4) 4

Ans.

Ans. Official answer NTA (2)

Question ID : 656445464

11. If $\alpha>\beta>\gamma>0$, then the expression $\cot^{-1}\left\{\beta+\frac{(1+\beta^2)}{(\alpha-\beta)}\right\}+\cot^{-1}\left\{\gamma+\frac{(1+\gamma^2)}{(\beta-\gamma)}\right\}+\cot^{-1}\left\{\alpha+\frac{(1+\alpha^2)}{(\gamma-\alpha)}\right\}$ is equal to :

(1) 0 (2) π (3) $\frac{\pi}{2}-(\alpha+\beta+\gamma)$ (4) 3π

Ans. Official answer NTA (2)

Sol.

Question ID : 656445465

12. Let $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=\vec{a}\times\left(\hat{i}-2\hat{k}\right)$ and $\vec{c}=\vec{b}\times\hat{k}$. Then the projection of $\vec{c}-2\hat{j}$ on $\vec{a}$ is :

(1) $\sqrt{14}$ (2) $2\sqrt{7}$ (3) $2\sqrt{14}$ (4) $3\sqrt{7}$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445469

13. The area of the region enclosed by the curves $y=e^x$, $y=|e^x-1|$ and y-axis is :

(1) $\log_e 2$ (2) $1+\log_e 2$ (3) $1-\log_e 2$ (4) $2\log_e 2-1$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445455

14. If the system of equations $x+2y-3z=2$
$2x+\lambda y+5z=5$
$14x+3y+\mu z=33$
has infinitely many solutions, then $\lambda+\mu$ is equal to : :

(1) 13 (2) 12 (3) 10 (4) 11

Ans. Official answer NTA (2)

Sol.

Question ID : 656445459

15. Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If $2A=5B$, then n is equal to :

(1) 21 (2) 20 (3) 19 (4) 22

Ans. Official answer NTA (1)

Sol.

Question ID : 656445463

16. The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is (3, 1) is :

(1) $48x + 25y = 169$ (2) $25x + 101y = 176$ (3) $5x + 16y = 31$ (4) $4x + 122y = 134$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445460

17. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to :

(1) 8925 (2) 8750 (3) 8575 (4) 9100

Ans. Official answer NTA (1)

Sol.

Question ID : 656445462

18. If the equation of the parabola with vertex $V\left(\frac{3}{2},3\right)$ and the directrix $x + 2y = 0$ is $\alpha x^2 + \beta y^2 - \gamma xy - 30x - 60y + 225 = 0$, then $\alpha + \beta + \gamma$ is equal to :

(1) 6 (2) 7 (3) 9 (4) 8

Ans. Official answer NTA (3)

Sol.

Question ID : 656445451

19. The function $f : (-\infty, \infty) \to (-\infty, 1)$, defined by $f(x)=\frac{2^x-2^{-x}}{2^x+2^{-x}}$ is :

(1) Both one-one and onto (2) Neither one-one nor onto

(3) One-one but not onto (4) Onto but not one-one

Ans. Official answer NTA (3)

Sol.

Question ID : 656445468

20. Let (2, 3) be the largest open interval in which the function $f(x) = 2\log_e(x-2) - x^2 + ax + 1$ is strictly increasing and (b, c) be the largest open interval, in which the function $g(x) = (x-1)^3(x+2-a)^2$ is strictly decreasing. Then $100(a+b-c)$ is equal to :

(1) 420 (2) 280 (3) 360 (4) 160

Ans. Official answer NTA (3)

Sol.

SECTION – B

Question ID : 656445473

21. If $\int \frac{2x^2+5x+9}{\sqrt{x^2+x+1}}dx = x\sqrt{x^2+x+1} + \alpha\sqrt{x^2+x+1} + \beta\log_e\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| + C$, where C is the constant of integration, then $\alpha + 2\beta$ is equal to _____.

Ans. Official answer NTA (16)

Sol.

Question ID : 656445472

22. Let P be the image of the point Q(7, –2, 5) in the line $L : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ and R(5, p, q) be a point on L, Then the square of the area of ΔPQR is _____.

Ans. Official answer NTA (957)

Sol.

Question ID : 656445475

23. Let $H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $H_2 : -\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their ecentricities be $e_1 = \sqrt{\frac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to _____.

Ans. Official answer NTA (55)

Sol.

Question ID : 656445474

24. Let y = y(x) be the solution of the differential equation $2\cos x \frac{dy}{dx} = \sin 2x - 4y\sin x$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{3}\right) = 0$, then $y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right)$ is equal to _____.

Ans. Official answer NTA (1)

Sol.

Question ID : 656445471

25. Number of functions $f : \{1, 2, ..., 100\} \to \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to _____.

Ans. Official answer NTA (392)

Sol.