# JEE Main January 2025

## Question Paper With Text Solution

## 28 January | Shift-1

# MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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# JEE MAIN JANUARY 2025 | 28TH JANUARY SHIFT-1

## SECTION – A

Question ID : 7364751519

1. Let $f : R \to R$ be a function defined by

$f(x) = (2+3a)x^2 + \left(\frac{a+2}{a-1}\right)x + b, a \neq 1$. If $f(x+y) = f(x) + f(y) + 1 - \frac{2}{7}xy$, then the value of $28\sum_{i=1}^{5}|f(i)|$ is :

(1) 545 (2) 675 (3) 735 (4) 715

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 7364751502

2. If $f(x) = \frac{2^x}{2^x + \sqrt{2}}, x \in \mathbb{R}$, then $\sum_{k=1}^{81} f\left(\frac{k}{82}\right)$ is equal to :

(1) 82 (2) $\frac{81}{2}$ (3) $81\sqrt{2}$ (4) 41

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 7364751515

3. If the image of the point (4, 4, 3) in the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{3}$ is $(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma$ is equal to :

(1) 12 (2) 8 (3) 7 (4) 9

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 7364751506

4. Let $T_r$ be the $r^{th}$ term of an A.P. If for some m, $T_m = \frac{1}{25}, T_{25} = \frac{1}{20}$, and $20\sum_{r=1}^{25} T_r = 13$, then $5m\sum_{r=m}^{2m} T_r$ is equal to :

(1) 126 (2) 142 (3) 98 (4) 112

**Ans.** Official answer NTA (1)

**Sol.**

Question ID : 7364751520

5. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96x^2 \cos^2 x}{\left(1+e^x\right)} dx = \pi\left(\alpha\pi^2 + \beta\right)$, $\alpha, \beta \in Z$ then $(\alpha + \beta)^2$ equals :

(1) 196 (2) 64 (3) 144 (4) 100

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 7364751501

6. The relation R = {(x, y) : x, y ∈ Z and x + y is even} is :

(1) symmetric and transitive but not reflexive (2) an equivalence relation

(3) reflexive and symmetric but not transitive (4) reflexive and transitive but not symmetric

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 7364751509

7. Two number $k_1$ and $k_2$ are randomly chosen from the set of natural numbers. Then, the probability that the value of $i^{k_1} + i^{k_2}, \left(i = \sqrt{-1}\right)$ is non-zero, equals :

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $\frac{2}{3}$ (4) $\frac{1}{4}$

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 7364751512

8. Let the equation of the circle, which touches x-axis at the point (a, 0), a > 0 and cuts off an intercept of length b on y-axis be $x^2 + y^2 - \alpha x + \beta y + \gamma = 0$. If the circle lies below x-axis, then the ordered pair $(2a, b^2)$ is equal to :

(1) $(\alpha, \beta^2 - 4\gamma)$ (2) $(\gamma, \beta^2 - 4\alpha)$ (3) $(\gamma, \beta^2 + 4\alpha)$ (4) $(\alpha, \beta^2 + 4\gamma)$

**Ans.** Official answer NTA (1)

**Sol.**

Question ID : 7364751507

9. The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, such that the sum of their first and last digits should not be more than 8 , is :

(1) 5720 (2) 4608 (3) 4607 (4) 5719

**Ans.** Official answer NTA (3)

**Sol.**

Question ID : 7364751517

10. The area (in sq. units) of the region $\{(x, y) : 0 \le y \le 2|x| + 1, 0 \le y \le x^2 + 1, |x| \le 3\}$ is :

(1) $\frac{64}{3}$ (2) $\frac{17}{3}$ (3) $\frac{32}{3}$ (4) $\frac{80}{3}$

**Ans.**

**Ans.** Official answer NTA (1)

Question ID : 7364751511

11. Let ${}^nC_{r-1} = 28$, ${}^nC_r = 56$ and ${}^nC_{r+1} = 70$. Let A (4cost, 4sint), B (2sint, –2cost) and C $(3r - n, r^2 - n - 1)$ be the vertices of a triangle ABC, where t is a parameter. If $(3x - 1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangle ABC, then $\alpha$ equals :

(1) 6 (2) 20 (3) 18 (4) 8

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 7364751518

12. Let for some function $y = f(x)$, $\int_0^x tf(t)dt = x^2 f(x)$, $x > 0$ and $f(2) = 3$. Then $f(6)$ is equal to :

(1) 1 (2) 3 (3) 6 (4) 2

**Ans.** Official answer NTA (1)

**Sol.**

Question ID : 7364751504

13. The sum, of the squares of all the roots of the equation $x^2 + |2x - 3| - 4 = 0$, is :

(1) $3\left(3-\sqrt{2}\right)$ (2) $6\left(3-\sqrt{2}\right)$ (3) $6\left(2-\sqrt{2}\right)$ (4) $3\left(2-\sqrt{2}\right)$

**Ans.** Official answer NTA (3)

**Sol.**

Question ID : 7364751514

14. Let A (x, y, z) be a point in xy-plane, which is equidistant from three points (0, 3, 2), (2, 0, 3) and (0, 0, 1). Let B = (1, 4, –1) and C = (2, 0, –2). Then among the statements

(S1) : $\Delta ABC$ is an isosceles right angled triangle, and

(S2) : the area of $\Delta ABC$ is $\frac{9\sqrt{2}}{2}$, :

(1) both are true (2) both are false (3) only (S2) is true (4) only (S1) is true

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 7364751516

15. The sum of all local minimum values of the function

$$f(x)=\begin{cases} 1-2x, & x<-1 \\ \frac{1}{3}(7+2|x|), & -1\le x\le 2 \\ \frac{11}{18}(x-4)(x-5), & x>2 \end{cases}$$

is :

(1) $\frac{171}{72}$ (2) $\frac{131}{72}$ (3) $\frac{167}{72}$ (4) $\frac{157}{72}$

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 7364751505

16. Let $<a_n>$ be a sequence such that $a_0 = 0$, $a_1 = \frac{1}{2}$ and $2a_{n+2} = 5a_{n+1} - 3a_n$, $n = 0, 1, 2, 3, .....$ Then $\sum_{k=1}^{100} a_k$ is equal to :

(1) $3a_{99} - 100$ (2) $3a_{99} + 100$ (3) $3a_{100} - 100$ (4) $3a_{100} + 100$

**Ans.** Official answer NTA (3)

**Sol.**

Question ID : 7364751508

17. Three defective oranges are accidently mixed with seven good ones and on looking at them, it is not possible to differentiate between them. Two oranges are drawn at random from the lot. If x denote the number of defective oranges, then the variance of x is :

(1) $\frac{26}{75}$ (2) $\frac{28}{75}$ (3) $\frac{18}{25}$ (4) $\frac{14}{25}$

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 7364751503

18. Let O be the origin, the point A be $z_1 = \sqrt{3} + 2\sqrt{2}i$, the point B($z_2$) be such that $\sqrt{3}|z_2| = |z_1|$ and $\arg(z_2) = \arg(z_1) + \frac{\pi}{6}$. Then :

(1) ABO is an obtuse angled isosceles triangle (2) ABO is a scalene triangle

(3) area of triangle ABO is $\frac{11}{4}$ (4) area of triangle ABO is $\frac{11}{\sqrt{3}}$

**Ans.** Official answer NTA (1)

**Sol.**

Question ID : 7364751510

19. Let ABCD be a trapezium whose vertices lie on the parabola $y^2 = 4x$. Let the sides AD and BC of the trapezium be parallel to y-axis. If the diagonal AC is of length $\frac{25}{4}$ and it passes through the point (1,0), then the area of ABCD is :

(1) $\frac{75}{8}$ (2) $\frac{125}{8}$ (3) $\frac{25}{2}$ (4) $\frac{75}{4}$

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 7364751513

20. $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$ is equal to :

(1) 1 (2) 0 (3) $\frac{33}{65}$ (4) $\frac{32}{65}$

**Ans.** Official answer NTA (2)

**Sol.**

**SECTION – B**

Question ID : 7364751521

21. Let M denote the set of all real matrices of order $3 \times 3$ and let $S = \{-3, -2, -1, 1, 2\}$. Let

$S_1 = \{A = [a_{ij}] \in M : A = A^T \text{ and } a_{ij} \in S, \forall\, i, j\}$,

$S_2 = \{A = [a_{ij}] \in M : A = -A^T \text{ and } a_{ij} \in S, \forall\, i, j\}$,

$S_3 = \{A = [a_{ij}] \in M : a_{11} + a_{22} + a_{33} = 0 \text{ and } a_{ij} \in S, \forall\, i, j\}$,

If $n(S_1 \cup S_2 \cup S_3) = 125\alpha$, then $\alpha$ equals _____.

**Ans.** Official answer NTA (1613)

**Sol.**

Question ID : 7364751525

22. Let $f(x) = \begin{cases} 3x, & x < 0 \\ \min\{1 + x + [x], x + 2[x]\}, & 0 \le x \le 2 \\ 5, & x > 2, \end{cases}$

where [.] denotes greatest integer function. If $\alpha$ and $\beta$ are the number of points, where f is not continuous and is not differentiable, respectively, then $\alpha + \beta$ equals _____.

**Ans.** Official answer NTA (5)

**Sol.**

Question ID : 7364751522

23. If $\alpha = 1 + \sum_{r=1}^{6} (-3)^{r-1}\, {}^{12}C_{2r-1}$, then the distance of the point $\left(12, \sqrt{3}\right)$ from the line $\alpha x - \sqrt{3}y + 1 = 0$ is _____.

**Ans.** Official answer NTA (5)

**Sol.**

Question ID : 7364751524

24. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=2\hat{i}+2\hat{j}+\hat{k}$ and $\vec{d}=\vec{a}\times\vec{b}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|, |\vec{c}-2\vec{a}|^2=8$ and the angle between $\vec{d}$ and $\vec{c}$ is $\frac{\pi}{4}$, then $|10-3\vec{b}\cdot\vec{c}|+|\vec{d}\times\vec{c}|^2$ is equal to _____.

**Ans.** Official answer NTA (6)

**Sol.**

Question ID : 7364751523

25. Let $E_1: \frac{x^2}{9}+\frac{y^2}{4}=1$ be an ellipse. Ellipses $E_i$'s are constructed such that their centres and eccentricities are same as that of $E_1$, and the length of minor axis of $E_i$ is the length of major axis of $E_{i+1}$ $(i \geq 1)$. If $A_i$ is the area of the ellipse $E_i$, then $\frac{5}{\pi}\left(\sum_{i=1}^{\infty} A_i\right)$, is equal to _____.

**Ans.** Official answer NTA (54)

**Sol.**