JEE Main January 2025

Question Paper With Text Solution

28 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2025 | 28TH JANUARY SHIFT-2

SECTION – A

Question ID : 656445694

1. If $f(x)=\int \frac{1}{x^{\frac{1}{4}}\left(1+x^{\frac{1}{4}}\right)}dx$, $f(0)=-6$, then $f(1)$ is equal to :

(1) $2-\log_e 2$ (2) $4(\log_e 2-2)$ (3) $\log_e 2+2$ (4) $4(\log_e 2+2)$

Ans. Official answer NTA (2)

Sol.

Question ID : 656445687

2. If A and B are the points of intersection of the circle $x^2+y^2-8x=0$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ and a point P moves on the line $2x-3y+4=0$, then the centroid of ΔPAB lies on the line :

(1) $x+9y=36$ (2) $6x-9y=20$ (3) $4x-9y=12$ (4) $9x-9y=32$

Ans. Official answer NTA (2)

Sol.

Question ID : 656445684

3. Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability, that the ball is drawn from Bag B_2, is :

(1) $\frac{2}{5}$ (2) $\frac{1}{3}$ (3) $\frac{4}{15}$ (4) $\frac{2}{3}$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445689

4. Let A, B, C be three points in xy-plane, whose position vector are given by $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $a\hat{i}+(1-a)\hat{j}$ respectively with respect to the origin O. If the distance of the point C from the line bisecting the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $\frac{9}{\sqrt{2}}$, then the sum of all the possible values of a is :

(1) 0 (2) 1 (3) 2 (4) $\frac{9}{2}$

Ans. Official answer NTA (2)

Sol.

Question ID : 656445678

5. Let $f : R - \{0\} \to (-\infty, 1)$ be a polynomial of degree 2, satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$. If f(K) = –2 K, then the sum of squares of all possible values of K is :

(1) 9 (2) 7 (3) 1 (4) 6

Ans. Official answer NTA (4)

Sol.

Question ID : 656445676

6. Let [x] denote the greatest integer less than or equal to x. Then the domain of $f(x) = \sec^{-1}(2[x] + 1)$ is :

(1) $(-\infty, -1] \cup [0, \infty)$ (2) $(-\infty, -1] \cup [1, \infty)$ (3) $(-\infty, \infty) - \{0\}$ (4) $(-\infty, \infty)$

Ans. Official answer NTA (4)

Sol.

Question ID : 656445688

7. If $\sum_{r=1}^{13}\left\{\frac{1}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}\right\} = a\sqrt{3}+b$, a, b ∈ Z, then $a^2 + b^2$ is equal to :

(1) 2 (2) 4 (3) 8 (4) 10

Ans. Official answer NTA (3)

Sol.

Question ID : 656445677

8. Let $f:[0,3] \to A$ be defined by $f(x) = 2x^3 - 15x^2 + 36x + 7$ and $g:[0,\infty) \to B$ be defined by $g(x) = \frac{x^{2025}}{x^{2025}+1}$.

If both the functions are onto and $S = \{x \in Z : x \in A \text{ or } x \in B\}$, then n(S) is equal to :

(1) 30 (2) 31 (3) 36 (4) 29

Ans. Official answer NTA (1)

Sol.

Question ID : 656445693

9. Let f be a real valued continuous function defined on the positive real axis such that $g(x) = \int_0^x tf(t)dt$. If $g(x^3) = x^6 + x^7$, then value of $\sum_{r=1}^{15} f(r^3)$ is :

(1) 340 (2) 320 (3) 270 (4) 310

Ans. Official answer NTA (4)

Sol.

Question ID : 656445679

10. If $\alpha + i\beta$ and $\gamma + i\delta$ are the roots of $x^2 - (3 - 2i)x - (2i - 2) = 0$, $i = \sqrt{-1}$, then $\alpha\gamma + \beta\delta$ is equal to :

(1) –2 (2) 2 (3) –6 (4) 6

Ans.

Ans. Official answer NTA (2)

Question ID : 656445681

11. For positive integers n, if $4a_n = (n^2 + 5n + 6)$ and $S_n = \sum_{k=1}^{n}\left(\frac{1}{a_k}\right)$, then the value of $507S_{2025}$ is :

(1) 675 (2) 1350 (3) 540 (4) 135

Ans. Official answer NTA (1)

Sol.

Question ID : 656445692

12. Let $f: R \to R$ be a twice differentiable function such that $f(2) = 1$. If $F(x) = x f(x)$ for all $x \in R$, $\int_0^2 x F'(x)\,dx = 6$ and $\int_0^2 x^2 F''(x)\,dx = 40$, then $F'(2) + \int_0^2 F(x)\,dx$ is equal to :

(1) 11 (2) 15 (3) 13 (4) 9

Ans. Official answer NTA (1)

Sol.

Question ID : 656445683

13. Let S be the set of all the words that can be formed by arranging all the letters of the word GARDEN. From the set S, one word is selected at random. The probability that the selected word will NOT have vowels in alphabetical order is :

(1) $\frac{2}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Ans. Official answer NTA (3)

Sol.

Question ID : 656445685

14. Two equal sides of an isosceles triangle are along $-x + 2y = 4$ and $x + y = 4$. If m is the slope of its third side, then the sum, of all possible distinct values of m, is :

(1) $-2\sqrt{10}$ (2) 12 (3) 6 (4) –6

Ans. Official answer NTA (3)

Sol.

Question ID : 656445695

15. The area of the region bounded by the curves $x(1 + y^2) = 1$ and $y^2 = 2x$ is :

(1) $2\left(\frac{\pi}{2} - \frac{1}{3}\right)$ (2) $\frac{1}{2}\left(\frac{\pi}{2} - \frac{1}{3}\right)$ (3) $\frac{\pi}{4} - \frac{1}{3}$ (4) $\frac{\pi}{2} - \frac{1}{3}$

Ans. Official answer NTA (4)

Sol.

Question ID : 656445680

16. Let $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ and $P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, $\theta>0$. If $B=PAP^T$, $C=P^TB^{10}P$ and the sum of the diagonal elements of C is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is :

(1) 2049 (2) 258 (3) 65 (4) 127

Ans. Official answer NTA (3)

Sol.

Question ID : 656445691

17. The square of the distance of the point $\left(\frac{15}{7},\frac{32}{7},7\right)$ from the line $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ in the direction of the vector $\hat{i}+4\hat{j}+7\hat{k}$ is :

(1) 54 (2) 44 (3) 66 (4) 41

Ans. Official answer NTA (3)

Sol.

Question ID : 656445690

18. If the components of $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ along and perpendicular to $\vec{b}=3\hat{i}+\hat{j}-\hat{k}$ respectively, are $\frac{16}{11}\left(3\hat{i}+\hat{j}-\hat{k}\right)$ and $\frac{1}{11}\left(-4\hat{i}-5\hat{j}-17\hat{k}\right)$, then $\alpha^2+\beta^2+\gamma^2$ is equal to :

(1) 16 (2) 18 (3) 26 (4) 23

Ans. Official answer NTA (3)

Sol.

Question ID : 656445686

19. If the midpoint of a chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is $\left(\sqrt{2},\frac{4}{3}\right)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is :

(1) 26 (2) 22 (3) 18 (4) 20

Ans. Official answer NTA (2)

Sol.

Question ID : 656445682

20. Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $\left(\sqrt[4]{3}+\sqrt[3]{4}\right)^{12}$. Then p + q is equal to :

(1) 283 (2) 287 (3) 295 (4) 299

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 656445700

21. If $y = y(x)$ is the solution of the differential equation, $\sqrt{4-x^2}\frac{dy}{dx}=\left(\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2-y\right)\sin^{-1}\left(\frac{x}{2}\right), -2\le x\le 2,$

$y(2)=\frac{\pi^2-8}{4}$, then $y^2(0)$ is equal to _____.

Ans. Official answer NTA (4)

Sol.

Question ID : 656445699

22. Let $f(x)=\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n}\left(\dfrac{\tan\left(\dfrac{x}{2^{r+1}}\right)+\tan^3\left(\dfrac{x}{2^{r+1}}\right)}{1-\tan^2\left(\dfrac{x}{2^{r+1}}\right)}\right)$. Then $\lim\limits_{x\to0}\dfrac{e^x-e^{f(x)}}{\left(x-f(x)\right)}$ is equal to _____.

Ans. Official answer NTA (1)

Sol.

Question ID : 656445696

23. The interior angles of a polygon with n sides, are in an A.P. with common difference 6°. If the largest interior angle of the polygon is 219°, then n is equal to _____.

Ans. Official answer NTA (20)

Sol.

Question ID : 656445697

24. The number of natural numbers, between 212 and 999 , such that the sum of their digits is 15, is _____.

Ans. Official answer NTA (64)

Sol.

Question ID : 656445698

25. Let A and B be the two points of intersection of the line $y+5=0$ and the mirror image of the parabola $y^2=4x$ with respect to the line $x+y+4=0$. If d denotes the distance between A and B, and a denotes the area of ΔSAB, where S is the focus of the parabola $y^2=4x$, then the value of $(a+d)$ is _____.

Ans. Official answer NTA (14)

Sol.