JEE Main January 2025

Question Paper With Text Solution

29 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2025 | 29TH JANUARY SHIFT-1

SECTION – A

Question ID : 656445531

1. Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :

(1) 90 (2) 122 (3) 84 (4) 108

Ans. Official answer NTA (1)

Sol.

Question ID : 656445534

2. Let $x_1, x_3, ...x_{10}$ be ten observations such that $\sum_{i=1}^{10}(x_i-2)=30, \sum_{i=1}^{10}(x_i-\beta)^2=98, \beta>2$ and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1-1)+4\beta, 2(x_2-1)+4\beta.....,2(x_{10}-1)+4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal :

(1) 110 (2) 90 (3) 120 (4) 100

Ans. Official answer NTA (4)

Sol.

Question ID : 656445528

3. The number of solutions of equation $\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0$ is :

(1) 1 (2) 4 (3) 2 (4) 3

Ans. Official answer NTA (2)

Sol.

Question ID : 656445543

4. The integral $80\int_0^{\pi/4}\left(\frac{\sin\theta+\cos\theta}{9+16\sin 2\theta}\right)d\theta$ is equal to :

(1) $6\log_e 4$ (2) $2\log_e 3$ (3) $3\log_e 4$ (4) $4\log_e 3$

Ans. Official answer NTA (4)

Sol.

Question ID : 656445540

5. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+7\hat{j}+3\hat{k}$ .Let $L_1:\vec{r}=\left(-\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\vec{a},\lambda\in R$ and $L_2:\vec{r}=\left(\hat{j}+\hat{k}\right)+\mu\vec{b},\ \mu\in R$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_2, and is parallel to $\vec{a}+\vec{b}$, then L_3 passes through the point:

(1) (8, 26, 12) (2) (5, 17, 4) (3) (–1, –1, 1) (4) (2, 8, 5)

Ans. Official answer NTA (1)

Sol.

Question ID : 656445532

6. Let P be the set of seven digit numbers with sum of their digits to 11. If the numbers in P are formed by using the digits 1, 2 and 3 only, then number of elements in the set P is :

(1) 173 (2) 164 (3) 158 (4) 161

Ans. Official answer NTA (4)

Sol.

Question ID : 656445529

7. Let $A=\left[a_{ij}\right]=\begin{bmatrix}\log_5 128 & \log_4 5\\ \log_5 8 & \log_4 25\end{bmatrix}$. If A_{ij} is the cofactor of $a_{ij}, C_{ij}=\sum_{k=1}^{2}a_{ik}A_{jk}, 1\le i,j\le 2$ and $C=[C_{ij}]$ then $8|C|$ is equal to :

(1) 288 (2) 222 (3) 262 (4) 242

Ans. Official answer NTA (4)

Sol.

Question ID : 656445533

8. The least value of n for which the number of integral terms in the Binomial expansion of $\left(\sqrt[3]{7}+\sqrt[12]{11}\right)^n$ is 183, is :

(1) 2148 (2) 2196 (3) 2172 (4) 2184

Ans. Official answer NTA (4)

Sol.

Question ID :656445530

9. The value of $\lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}\right)$ is :

(1) 2 (2) 7/3 (3) 4/3 (4) 5/3

Ans. Official answer NTA (4)

Sol.

Question ID : 656445526

10. Define a relation R on the interval $\left[0,\frac{\pi}{2}\right)$ by x R y if and only if $\sec^2 x - \tan^2 y = 1$. Then R is :

(1) an equivalence relation

(2) reflexive but neither symmetric not transitive

(3) both reflexive and transitive but not symmetric

(4) both reflexive and symmetric but not transitive

Ans. Official answer NTA (1)

Sol.

Question ID : 656445544

11. Let the area of the region $\left\{(x,y): 2y \le x^2+3, y+|x|\le 3, y \ge |x-1|\right\}$ be A. Then 6A is equal to :

(1) 16 (2) 18 (3) 12 (4) 14

Ans. Official answer NTA (4)

Sol.

Question ID : 656445539

12. Let $L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$ and $L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$ be two lines. Let L_3 be a line passing through the point (α, β, γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha - 11\beta - 8\gamma|$ is equals :

(1) 16 (2) 25 (3) 20 (4) 18

Ans. Official answer NTA (2)

Sol.

Question ID : 656445545

13. Let $y = y(x)$ be the solution of the differential equation

$\cos x\left(\log_e(\cos x)\right)^2 dy + \left(\sin x - 3y \sin x \log_e(\cos x)\right)dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{4}\right) = \frac{-1}{\log_e 2}$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $-\frac{1}{\log_e(4)}$ (2) $\frac{2}{\log_e(3) - \log_e(4)}$ (3) $\frac{1}{\log_e(4) - \log_e(3)}$ (4) $\frac{1}{\log_e(3) - \log_e(4)}$

Ans. Official answer NTA (4)

Sol.

Question ID : 656445536

14. Let the line $x + y = 1$ meet the circle $x^2 + y^2 = 4$ at the points A and B. If the line perpendicular to AB and passing through the mid point of the chord AB intersects the circle at C and D, then the area of the quadrilateral ADBC is equal to :

(1) $3\sqrt{7}$ (2) $5\sqrt{7}$ (3) $\sqrt{14}$ (4) $2\sqrt{14}$

Ans. Official answer NTA (4)

Sol.

Question ID : 656445542

15. Let M and m respectively be the maximum and minimum values of

$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix},\ x\in R.$$ Then M^4-m^4 is equal to :

(1) 1040 (2) 1295 (3) 1215 (4) 1280

Ans. Official answer NTA (4)

Sol.

Question ID :656445537

16. Two parabolas have the same focus (4, 3) and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then $(AB)^2$ is equal to :

(1) 96 (2) 192 (3) 392 (4) 384

Ans. Official answer NTA (2)

Sol.

Question ID : 656445538

17. Let the ellipse $E_1:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$ and $E_2:\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$, $A<B$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet A,B,C and D, then the area of the quadrilateral ABCD equals :

(1) $\frac{24\sqrt{6}}{5}$ (2) $6\sqrt{6}$ (3) $\frac{12\sqrt{6}}{5}$ (4) $\frac{18\sqrt{6}}{5}$

Ans. Official answer NTA (1)

Sol.

Question ID : 656445535

18. Let ABC be a triangle formed by the lines $7x-6y+3=0$, $x+2y-31=0$ and $9x-2y-19=0$. Let the point (h, k) be the image of the centroid of ΔABC in the line $3x+6y-53=0$. Then h^2+k^2+hk is equal to:

(1) 47 (2) 37 (3) 40 (4) 36

Ans. Official answer NTA (2)

Sol.

Question ID : 656445527

19. Let $|z_1-8-2i| \le 1$ and $|z_2-2+6i| \le 2, z_1, z_2 \in C$. Then minimum value of $|z_1-z_2|$ is :

(1) 7 (2) 3 (3) 10 (4) 13

Ans. Official answer NTA (1)

Sol.

Question ID : 656445541

20. Let $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$, $\vec{b}=3\hat{i}-5\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}=\vec{c}\times\vec{b}$ and $(\vec{a}+\vec{c}).(\vec{b}+\vec{c})=168$. Then the maximum value of $|\vec{c}|^2$ is :

(1) 308 (2) 154 (3) 77 (4) 462

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 656445549

21. Let [t] be the greatest integer less than or equal to t. Then least value of $p \in N$ for which $\lim\limits_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+...+\left[\frac{9^2}{x^2}\right]\right)\right)\ge 1$ is equal to _____.

Ans. Official answer NTA (24)

Sol.

Question ID : 656445548

22. Let $S=\{x:\cos^{-1}x=\pi+\sin^{-1}x+\sin^{-1}(2x+1)\}$. Then $\sum_{x\in S}(2x-1)^2$ is equal to _____.

Ans. Official answer NTA (5)

Sol.

Question ID : 656445547

23. The number of 6-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is _____.

Ans. Official answer NTA (1405)

Sol.

Question ID : 656445546

24. Let $S=\left\{m\in Z:A^{m^2}+A^m=3I-A^{-6}\right\}$, where $A=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$. Then n(S) is equal to _____.

Ans. Official answer NTA (2)

Sol.

Question ID : 656445550

25. Let $f:(0,\infty)\to R$ be a twice differentiable function. If for some $a\neq 0, \int_0^1 f(\lambda x)d\lambda = af(x), f(1)=0$ and $f(16)=\frac{1}{8}$, then $16-f'\left(\frac{1}{16}\right)$ is equal to _____.

Ans. Official answer NTA (112)

Sol.