JEE Main January 2025

Question Paper With Text Solution

29 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2025 | 29TH JANUARY SHIFT-2

SECTION – A

Question ID : 6564451132

1. The remainder, when 7^{103} is divided by 23, is equal to :

(1) 17 (2) 6 (3) 9 (4) 14

Ans. Official answer NTA (4)

Sol.

Question ID : 6564451138

2. If $\sin x + \sin^2 x = 1, x \in \left(0, \frac{\pi}{2}\right)$, then

$\left(\cos^{12} x + \tan^{12} x\right) + 3\left(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x\right) + \left(\cos^6 x + \tan^6 x\right)$ is equal to :

(1) 4 (2) 2 (3) 3 (4) 1

Ans. Official answer NTA (2)

Sol.

Question ID : 6564451133

3. Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains n white balls and 3 black. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability, that the ball drawn is white, is 29/45, then n is equal to :

(1) 3 (2) 5 (3) 4 (4) 6

Ans. Official answer NTA (4)

Sol.

Question ID : 6564451144

4. Let the area of enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ be α. If $9\alpha = \beta\pi + \gamma; \beta, \gamma$ are integers, then the value of $|\beta - \gamma|$ equals.

(1) 18 (2) 33 (3) 27 (4) 15

Ans. Official answer NTA (2)

Sol.

Question ID : 6564451136

5. Let a circle C pass through the points (4, 2) and (0, 2), and its centre lie on $3x + 2y + 2 = 0$. Then the length of the chord, of the circle C, whose mid-point is (1, 2), is :

(1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Ans. Official answer NTA (1)

Sol.

Question ID : 6564451129

6. Let $A = [a_{ij}]$ be a matrix of order 3×3, with $a_{ij} = \left(\sqrt{2}\right)^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha + \beta\sqrt{2}, \alpha, \beta \in Z$, then $\alpha + \beta$ is equal to :

(1) 168 (2) 210 (3) 224 (4) 280

Ans. Official answer NTA (3)

Sol.

Question ID : 6564451141

7. Let P be the foot of the perpendicular from point (1, 2, 2) on the line $L : \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$. Let the line $\vec{r} = \left(-\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right), \lambda \in R$, intersect the line L, at Q. Then $2(PQ)^2$ is equal to :

(1) 19 (2) 27 (3) 25 (4) 29

Ans. Official answer NTA (2)

Sol.

Question ID : 6564451142

8. Let $f(x) = (x^2 - 1)|x^2 - ax + 2| + \cos|x|$ the function be not differentiable at the two points $x = \alpha = 2$ and $x = \beta$. Then the distance of the point (α, β) from the line $12x + 5y + 10 = 0$ is equal to :

(1) 4 (2) 5 (3) 2 (4) 3

Ans. Official answer NTA (4)
Answer by MATRIX (Bonus)

Sol.

Question ID : 6564451137

9. If $\alpha x+\beta y=109$ is the equation of the chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, whose mid point is $\left(\frac{5}{2},\frac{1}{2}\right)$, then $\alpha+\beta$ is equal to :

(1) 37 (2) 58 (3) 46 (4) 72

Ans. Official answer NTA (2)

Sol.

Question ID : 6564451145

10. If for solution curve y = f(x) of the differential equation $\frac{dy}{dx}+(\tan x)y=\frac{2+\sec x}{(1+2\sec x)^2}$, $x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right), f\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{10}$, then $f\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{4-\sqrt{2}}{14}$ (2) $\frac{5-\sqrt{3}}{2\sqrt{2}}$ (3) $\frac{9\sqrt{3}+3}{10\left(4+\sqrt{3}\right)}$ (4) $\frac{\sqrt{3}+1}{10\left(4+\sqrt{3}\right)}$

Ans.

Ans. Official answer NTA (2)

Question ID : 6564451135

11. Let the line x + y = 1 meet the axes of x and y at A and B, respectively. A right angled triangle AMN is inscribed in the triangle OAB, where O is the origin and the points M and N lie on the lines OB and AB respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and AN : NB = λ : 1, then the sum of all possible value(s) of is λ :

(1) 1/2 (2) 5/2 (3) 13/6 (4) 2

Ans. Official answer NTA (4)

Sol.

Question ID : 6564451128

12. If the set of all $a \in \mathbf{R}$, for which the equation $2x^2 + (a-5)x + 15 = 3a$ has no real roots, is the interval (α, β) and $X = \{x \in Z : \alpha < x < \beta\}$, then $\sum_{x \in X} x^2$ is euqal to :

(1) 2109 (2) 2119 (3) 2139 (4) 2129

Ans. Official answer NTA (3)

Sol.

Question ID : 6564451134

13. Let $A[a_{ij}]$ be a 2×2 matrix such that $a_{ij} \in \{0, 1\}$ for all i and j. Let the random variable X denote the possible values of the determinant of the matrix A. Then, the variance of X is :

(1) 5/8 (2) 3/4 (3) 1/4 (4) 3/8

Ans. Official answer NTA (4)

Sol.

Question ID : 6564451131

14. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440^{th} position in the arrangement is :

(1) PRNAUK (2) PRNAKU (3) PRKANU (4) PRKAUN

Ans. Official answer NTA (4)

Sol.

Question ID : 6564451143

15. Let $f(x) = \int_0^x t\left(t^2 - 9t + 20\right)dt, 1 \le x \le 5$. If the range of f is $[\alpha, \beta]$, then $4(\alpha + \beta)$ equals :

(1) 157 (2) 253 (3) 154 (4) 125

Ans. Official answer NTA (1)

Sol.

Question ID : 6564451126

16. If the domain of the function $\log_5\left(18x-x^2-77\right)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2+\beta^2+\gamma^2$ is equal to :

(1) 179 (2) 186 (3) 195 (4) 174

Ans. Official answer NTA (2)

Sol.

Question ID : 6564451127

17. Let $S = N \cup \{0\}$. Define a relation R from S to R by:

$$R = \left\{\left(x, y\right) : \log_e y = x \log_e\left(\frac{2}{5}\right), x \in S, y \in R\right\}.$$

Then, the sum of all elements in the range of R is equal to :

(1) 3/2 (2) 10/9 (3) 5/3 (4) 5/2

Ans. Official answer NTA (3)

Sol.

Question ID :6564451139

18. Let $\hat{a}$ be a unit vector perpendicular to the vectors $\vec{b} = \hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c} = 2\hat{i}-3\hat{j}+\hat{k}$, and makes an angle of $\cos^{-1}\left(-\frac{1}{3}\right)$ with vector $\hat{i}+\hat{j}+\hat{k}$. If $\hat{a}$ makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i}+\alpha\hat{j}+\hat{k}$, then the value of α is :

(1) $-\sqrt{3}$ (2) $\sqrt{3}$ (3) $\sqrt{6}$ (4) $-\sqrt{6}$

Ans. Official answer NTA (4)

Sol.

Question ID : 6564451140

19. Let a straight lien L pass through the point P (2, –1, 3) and be perpendicular to the lines $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-2}$ and $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z+2}{4}$. If the line L intersects the yz-plane at the point Q, then the distance between the points P and Q is :

(1) $2\sqrt{3}$ (2) 2 (3) 3 (4) $\sqrt{10}$

Ans. Official answer NTA (3)

Sol.

Question ID : 6564451130

20. Let α, β ($\alpha \neq \beta$) be the values of m, for which the equations x + y + z = 1; x + 2y + 4z = m and $x+4y+10z=m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}\left(n^{\alpha}+n^{\beta}\right)$ is equal to :

(1) 440 (2) 3080 (3) 3410 (4) 560

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 6564451149

21. If $\lim\limits_{t\to 0}\left(\int_0^1 (3x+5)^t\,dx\right)^{\frac{1}{t}}=\frac{\alpha}{5e}\left(\frac{8}{5}\right)^{\frac{2}{3}}$, then α is equal to _____.

Ans. Official answer NTA (64)

Sol.

Question ID : 6564451150

22. If $24\int_0^{\pi/4}\left(\sin\left|4x-\frac{\pi}{12}\right|+\left[2\sin x\right]\right)dx=2\pi+\alpha$, where [.] denotes the greatest integer function, then α is equal to _____.

Ans. Official answer NTA (12)

Sol.

Question ID : 6564451146

23. Let integers $a, b \in [-3, 3]$ be such that $a + b \neq 0$. Then the number of all possible ordered pairs (a, b), for which $\left|\frac{z-a}{z+b}\right| = 1$ and $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1, z \in C$, where ω and ω^2 are the roots of $x^2 + x + 1 = 0$, is equal to _____.

Ans. Official answer NTA (10)

Sol.

Question ID : 6564451148

24. Let $y^2 = 12x$ be the parabola and S be its focus. Let PQ be a focal chord of the parabola such that $(SP)(SQ = \frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2 + 64y^2 - \alpha x + -64\sqrt{3}y = \beta$, then $\beta - \alpha$ is equal to_____.

Ans. Official answer NTA (1328)

Sol.

Question ID : 6564451147

25. Let $a_1, a_2, ..., a_{2024}$ be an Arithmetic Progression such that $a_1 + (a_5 + a_{10} + a_{15} + ... + a_{2020}) + a_{2024} = 2233$. Then $a_1 + a_2 + a_3 + ...+a_{2024}$ is equal to _____.

Ans. Official answer NTA (11132)

Sol.